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Kawauchi's second duality and knotted surfaces in 4-sphere

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§1. Introduction

Let M be a compact connected oriented n-dimensional topological manifold with an integral cohomology class $\gamma \in H^1(M; \boldsymbol{Z})$ of infinite order. Then we have the infinite cyclic covering space $\tilde{M}$ over M associated with γ and the finitely generated Λ-modules

$$H_*(\tilde{M}) = H_*(\tilde{M}; \boldsymbol{Z}) \text{ and } H_*(\tilde{M}, \partial\tilde{M}) = H_*(\tilde{M}, \partial\tilde{M}; \boldsymbol{Z}).$$

Here Λ is the integral group ring of the infinite cyclic group generated by t, and t acts on these homology groups by the induced isomorphisms of the generator specified by γ of the covering transformation group.

Now, we recall the result of A. Kawauchi on these Λ-modules, by using the following notations for any finitely generated Λ-module H:

DH = the unique maximal finite Λ-submodule of H,

$\mathrm{tor}_\Lambda H$(resp. tor H) = the Λ-(resp. $\boldsymbol{Z}$-) torsion part of H,

$BH = H/\mathrm{tor}_\Lambda H$, $\mathrm{E}^i H = \mathrm{Ext}^i_\Lambda(H, \Lambda)$ = the i-th Ext-group over Λ.

THEOREM 1 (Kawauchi's second duality theorem [7]). *Let p and q be integers with $p + q = n - 2$. Then there exist t-anti Λ-epimorphisms*

$$\theta_p: DH_p(\tilde{M}) \longrightarrow \mathrm{E}^1BH_{q+1}(\tilde{M}, \partial\tilde{M}),\ \theta'_q: DH_q(\tilde{M}, \partial\tilde{M}) \longrightarrow \mathrm{E}^1BH_{p+1}(\tilde{M})$$

such that the finite Λ-submodules Ker θ_p *and* Ker θ'_q *are dual by a t-isometric, $(-1)^{pq+1}$-symmetric and non-singular pairing*

$$\ell: \mathrm{Ker}\ \theta_p \times \mathrm{Ker}\ \theta'_q \longrightarrow \boldsymbol{Q}/\boldsymbol{Z}\ (\boldsymbol{Q}: \textit{the rational number field}).$$

Moreover, this pairing is a proper oriented homotopy invariant.

In this paper, we study this pairing in a geometric way under the following assumption (*), and give some applications on knotted surfaces in S^4.

(*) For M and γ of above, assume that the Poincaré dual of γ in $H_{n-1}(M, \partial M)$ can be represented by a bicollared proper oriented $(n-1)$-dimensional submanifold V of M, which may be regarded as $V \subset \tilde{M}$.

Under this assumption, we have the linking pairing