HIROSHIMA MATH. J.
19 (1989), 575–585

Intersections of subideals of Lie algebras

Dedicated to the memory of Professor Sigeaki Tôgô

Masanobu HONDA

(Received January 4, 1989)

Introduction

The author [5], [6] and Nomura [11] have investigated the class $\mathfrak{L}^{\infty}$ of Lie algebras in which the join of any collection of subideals is always a subideal. On the other hand, concerning the class $\mathfrak{L}_{\infty}$ of Lie algebras in which the intersection of any collection of subideals is always a subideal, very little is known except the fact that $\mathfrak{M} \leq \mathfrak{L}_{\infty}$ ([5, Lemma 3.2]), where $\mathfrak{M}$ is the class of Lie algebras having an upper bound for the steps of all subideals. The purpose of this paper is to present further results concerning the class $\mathfrak{L}_{\infty}$ and investigate related classes.

In Section 2 we shall first prove that in any Lie algebra the intersection of any collection of descendant (resp. weakly descendant, serial, weakly serial) subalgebras is always descendant (resp. weakly descendant, serial, weakly serial) (Theorem 2.2). We shall secondly characterize the class $\mathfrak{L}_{\infty}$ as the class of Lie algebras in which every descendant subalgebra is a subideal (Theorem 2.3).

The group-theoretic analogue of the class $\mathfrak{M}$ is usually denoted by $\mathfrak{B}$. Robinson [12] has proved that if a group G has a normal subgroup N such that N has a composition series of finite length and G/N is in the class $\mathfrak{B}$, then G is in the class $\mathfrak{B}$. In Section 3 we shall prove that if a Lie algebra L has an ideal I such that I has a composition series of finite length and L/I is in the class $\mathfrak{L}_{\infty}$ (resp. $\mathfrak{L}_{\infty}$(asc), $\mathfrak{M}$, $\mathfrak{D}$(asc, si)), then L is in the class $\mathfrak{L}_{\infty}$ (resp. $\mathfrak{L}_{\infty}$(asc), $\mathfrak{M}$, $\mathfrak{D}$(asc, si)) (Theorem 3.4), where $\mathfrak{L}_{\infty}$(asc) is the class of Lie algebras in which the intersection of any collection of ascendant subalgebras is always ascendant, and $\mathfrak{D}$(asc, si) is the class of Lie algebras in which every ascendant subalgebra is a subideal.

In Section 4 we shall first prove that if a Lie algebra having an abelian ideal of codimension 1 is in the class $\mathfrak{L}_{\infty}$, then it must be in the class $\mathfrak{M}$ (Proposition 4.2). Secondly we shall present a sufficient condition for Lie algebras in the class $\mathfrak{L}_{\infty}$ to be nilpotent.

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Throughout this paper we always consider not necessarily finite-