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Periodic and almost periodic stability of solutions to degenerate parabolic equations

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Introduction

This paper is concerned with periodic and almost periodic behavior of solutions to the following problem:

$$
(0.1) \qquad \begin{cases} u' - \Delta v = f, \ v \in \beta(u), & \text{in } (0, \infty) \times \Omega, \\ v = g_0 \quad \text{on } (0, \infty) \times \Gamma_0, & \\ \partial_\nu v + p \cdot v = g_1 & \text{on } (0, \infty) \times (\Gamma \backslash \Gamma_0), \\ u(0, \cdot) = u_0 & \text{in } \Omega. \end{cases}
$$

Here $u' = (\partial/\partial t)u$, Ω is a bounded domain in $\mathbf{R}^N$ ($N \geq 1$) with smooth boundary Γ, Γ_0 is a measurable subset of Γ with positive surface measure, p is a non-negative bounded measurable function on Γ, ∂_ν denotes the outward normal derivative on Γ, and β is a maximal monotone graph in $\mathbf{R} \times \mathbf{R}$. Damlamian and Kenmochi have studied in [8, 9] the global behavior of solutions to (0.1) in the case in which β is Lipschitz continuous. The Lipschitz continuous case is effective for Stefan problems in weak (enthalpy) formulation, but it is in general required to assume that β is multi-valued. In fact, we do have this situation for instance in the weak formulations of free boundary problems arising from Hele-Shaw flows as well as electrochemical machining processes, see [5, 18, 19, 20]. As observed by Damlamian [6, 7], problem (0.1) is formulated as an evolution equation by means of time-dependent subdifferentials in an appropriate Hilbert space. In Kenmochi-Ôtani [14, 15], the periodic and almost periodic stability of solutions to a general class of evolution equations with time-dependent subdifferentials have been studied. However, it does not seem that their result is directly applicable to the problem (0.1) if both β and β^{-1} are multi-valued. In this paper, we extend a part of the result given in [9] to a class of maximal monotone graphs β so that the inverse of the Heaviside function may be contained. This is necessarry to treat the problems for Hele-Shaw flows and electrochemical machining processes, since in these cases β is the inverse of the Heaviside function. The main results of this paper were already announced in [17], and this paper contains their complete proofs.

We shall first establish existence theorems of periodic (resp. almost periodic) solutions of problem (0.1) and then discuss their asymptotic stability,