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Asymptotic and integral equivalence of multivalued differential systems

Dedicated to Professor Marko Švec on the occasion of his seventieth birthday

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The aim of this paper is to study the asymptotic and (ψ, p)-integral equivalence of differential systems of the form

$$x'(t) \in A(t)x(t) + F(t, x(t), Sx(t)), \tag{a}$$

$$y'(t) = A(t)y(t), \tag{b}$$

where $A(t)$ is an $n \times n$ matrix-function defined on $J = [0, \infty)$ whose elements are integrable on compact subsets of J; x and y are n-dimensional vectors, S is a continuous operator mapping the set $B_\psi(J)$ of continuous and ψ-bounded functions defined on J to $B_\psi(J)$ in the sense that if $x_n \xrightarrow{q} x$ then $Sx_n \xrightarrow{q} Sx$ (precise definitions are given below) e.g.

$$Sx(t) := \int_0^t K(t, s)x(s)\, ds,$$

under certain conditions on the function $K(t, s)$, and $F(t, u, v)$ is a nonempty, compact and convex subset of R^n for each $(t, u, v) \in J \times R^n \times R^n$.

By a solution of (a), we mean an absolutely continuous function $x(t)$ on some nondegenerate subinterval of J which satisfies (a) almost everywhere (a.e.).

DEFINITION 1 (A. Haščák and M. Švec [10]). Let $\psi(t)$ be a positive continuous function on an interval $[t_0, \infty)$ and let $p > 0$. We shall say that two systems (a) and (b) are (ψ, p)-integral equivalent on $[t_0, \infty)$ iff for each solution $x(t)$ of (a) there exists a solution $y(t)$ of (b) such that

$$\psi^{-1}(t)|x(t) - y(t)| \in L_p([t_0, \infty)) \tag{c}$$

and conversely, for each solution $y(t)$ of (b) there exists a solution $x(t)$ of (a) such that (c) holds.

By a restricted (ψ, p)-integral equivalence between (a) and (b) we shall mean that the relation (c) is satisfied for some subsets of solutions of (a) and (b), e.g. for the ψ-bounded solutions.