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On oscillation of linear neutral differential equations of higher order

Dedicated to Professor Marko Švec on the occasion of his seventieth birthday

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1. Introduction

We consider linear neutral functional differential equations of the form

$$\text{(A)} \qquad \frac{d^n}{dt^n}[x(t) - h(t)x(\tau(t))] + \sigma p(t)x(g(t)) = 0\,, \qquad t \geqq t_0\,,$$

where $n \geqq 2$, $\sigma = +1$ or -1 and the following conditions are assumed to hold without further mention:

(B) (a) $h : [t_0, \infty) \to (0, \infty)$ is continuous;
(b) $\tau : [t_0, \infty) \to \boldsymbol{R}$ is continuous and strictly increasing, and $\lim_{t\to\infty} \tau(t) = \infty$;
(c) $p : [t_0, \infty) \to (0, \infty)$ is continuous;
(d) $g : [t_0, \infty) \to \boldsymbol{R}$ is continuous and $\lim_{t\to\infty} g(t) = \infty$.

Our aim is to establish new oscillation criteria for equation (A), i.e., sufficient conditions under which all proper solutions of (A) are oscillatory. By a proper solution of (A) we here mean a continuous function $x : [t_x, \infty) \to \boldsymbol{R}$ such that $x(t) - h(t)x(\tau(t))$ is n-times continuously differentiable, $x(t)$ satisfies (A) for all sufficiently large $t \geqq t_x$ and $\sup \{|x(t)| : t \geqq T\} > 0$ for any $T \geqq t_x$. Such a solution is called oscillatory if it has arbitrarily large zeros in $[t_x, \infty)$ and it is called nonoscillatory otherwise.

The problem of oscillation of neutral functional differential equations has received considerable attention in the last few years (see, for example, the papers [1–14, 17–20] and the references cited therein). However, most of the works on the subject has been focused on first and second order equations with constant parameters and very little has been published on higher order neutral equations. For some particular results we refer to [7], [13–14] and [18].

The present paper is an attempt to make a systematic study of oscillatory properties of higher order equations of the form (A) with general arguments $h(t)$ and $\tau(t)$. Our technique here is based on deriving two infinite sequences $\{(\mathrm{I}_k^+, \sigma)\}_{k=0}^{\infty}$ and $\{(\mathrm{I}_m^-, \sigma)\}_{m=1}^{\infty}$ of "non-neutral" functional differential inequalities