HIROSHIMA MATH. J.
20 (1990), 399–405

Exactness and Bernoulliness of generalized random dynamical systems

Haruo TOTOKI and Yoshiki TSUJII

(Received August 29, 1989)

Recently the second author of the present paper proposed the idea of generalized random dynamical systems and investigated their ergodic properties in [3]. This paper is a supplementary note for [3]. We will investigate the exactness and the Bernoulliness of skew product transformations associated with generalized random dynamical systems.

§1. Preliminaries

Let $(S, \mathscr{B}, \mu)$ and $(Y, \mathscr{F}, \nu)$ be standard probability spaces. We consider $(S, \mathscr{B}, \mu)$ as a phase space and $(Y, \mathscr{F}, \nu)$ as a parameter space. Namely for each $y \in Y$ a measure-preserving transformation φ_y on $(S, \mathscr{B}, \mu)$ is given. We assume that the mapping $(s, y) \mapsto \varphi_y(s)$ is $\mathscr{B} \times \mathscr{F}/\mathscr{B}$-measurable. We are concerned with the behavior of the random orbit

$$X_n(s) = \varphi_{y_n} \circ \varphi_{y_{n-1}} \circ \cdots \circ \varphi_{y_1}(s), \qquad s \in S, \qquad y_1, \ldots, y_n \in Y, \qquad n \geq 1,$$

where $y_1, \ldots, y_n$ are taken randomly in the following manner. There are given a family of probability density functions $\{\gamma(s, y), s \in S\}$ on Y:

$$\gamma(s, y) \geq 0, \qquad \int_Y \gamma(s, y)\, d\nu(y) = 1, \qquad s \in S,$$

and a sub-σ-field $\mathscr{B}_0 \subset \mathscr{B}$ such that

(i) $\gamma(s, y)$ is $\mathscr{B}_0 \times \mathscr{F}$-measurable

and

(ii) $\varphi_y^{-1}\mathscr{B}$ and $\mathscr{B}_0$ are independent for each $y \in Y$.

Each y_k $(k \geq 1)$ is choosen according to the probability measure $\gamma(X_{k-1}(s), y)\, d\nu(y)$ where $X_0(s) = s$. Then $X = \{X_n(s), n \geq 0\}$ becomes a stationary Markov chain with the transition probability

$$P(s, B) = \int_Y 1_B(\varphi_y(s))\gamma(s, y)\, d\nu(y), \qquad B \in \mathscr{B},$$

and the stationary measure μ. Let T be the corresponding Markov operator:

$$Tf(s) = \int_S f(t)P(s, dt) = \int_Y f(\varphi_y(s))\gamma(s, y)\, d\nu(y), \qquad f \in L^1(S, \mu).$$