HIROSHIMA MATH. J.
20 (1990), 385–394

Nonnegative entire solutions of a class of degenerate semilinear elliptic equations

Nobuyoshi FUKAGAI

(Received August 1, 1989)

1. Introduction

This paper is concerned with the existence and qualitative behavior of nonnegative entire solutions of the degenerate elliptic equation

$$\text{(A)} \qquad \Delta(u^m) + u(1-u)(u-a) = 0\,, \qquad x \in R^n\,, \qquad n \geq 2\,,$$

where m and a are positive constants. By a radial entire solution of (A) is meant a function $u \in C(R^n)$ depending only on $|x|$ such that $u^m \in C^2(R^n)$ and that (A) is satisfied at every point of R^n.

The one-dimensional case of (A) has been studied by Aronson, Crandall and Peletier [1], who have shown, among other things, that (A) ($n = 1$) has nonnegative radial entire solutions u with compact support provided $m > 1$ and $0 < a < (m+1)/(m+3)$. Our purpose here is to extend some of the results of [1] to the higher dimensional case ($n \geq 2$) of (A) by proving the theorem below.

THEOREM. *Let $0 < a < (m+1)/(m+3)$. Then, there exists a constant $u_* \in (0, 1)$ such that* (A) *has a nonnegative radial entire solution $u(x)$ satisfying $u(0) = u_0$ if $0 < u_0 \leq u_*$, and* (A) *has no nonnegative entire solution $u(x)$ satisfying $u(0) = u_0$ if $u_* < u_0 < 1$. Furthermore, the following statements hold.*

(i) *If $0 < u_0 < u_*$, the radial entire solution $u(x)$ satisfying $u(0) = u_0$ oscillates around a and converges to a as $|x| \to \infty$.*

(ii) *The radial entire solution $u(x)$ satisfying $u(0) = u_*$ decreases monotonically to zero as $|x| \to \infty$. This solution has compact support if $m > 1$.*

The substitution $v = u^m$ reduces (A) to

$$\text{(B)} \qquad \Delta v + v^{1/m}(1 - v^{1/m})(v^{1/m} - a) = 0\,, \qquad x \in R^n\,, \qquad n \geq 2$$

which is formally a special case of the equation

$$\text{(C)} \qquad \Delta v + f(v) = 0\,, \qquad x \in R^n, \qquad n \geq 2\,.$$

Although there is a vast literature devoted to the investigation of (C) from various viewpoints (see e.g. [1–6, 13–18]), none of the existing results for (C) seems to be applicable to establish the existence of entire solutions of (B)