HIROSHIMA MATH. J.
20 (1990), 365–384

Homotopy commutativity of the loop space of a finite CW-complex

Hideyuki KACHI

(Received July 14, 1989)

0. Introduction

As a generalization of a topological group in the homotopy theory, an H-space (or a Hopf space) is defined to be a topological space Y with a base point $*$ admitting a continuous multiplication $\mu: Y \times Y \to Y$ such that $*$ acts as a two sided homotopy unit, that is, the restrictions $\mu | Y \times *$ and $\mu | * \times Y$ are both homotopic (preserving $*$) to the identity map $\mathrm{id}_Y: Y \to Y$; and an H-space $Y = (Y, \mu)$ is said to be homotopy associative when the two maps $\mu(\mu \times \mathrm{id}_Y)$ and $\mu(\mathrm{id}_Y \times \mu)$ of $Y \times Y \times Y$ to Y are homotopic. The loop space ΩX of a based space X admitting the usual loop multiplication is another important example; and in the homotopy theory, ΩX can be regarded as a topological group G when $X = B_G$ is the classifying space of G.

Moreover, as a generalization of a topological abelian group, an H-space $Y = (Y, \mu)$ is said to be homotopy commutative when $\mu: Y \times Y \to Y$ is homotopic to μT for the homeomorphism T on $Y \times Y$ commuting coordinates. A compact connected Lie group G is homotopy commutative if and only if G is abelian, that is, G is a torus, the product of some copies of the circle group S^1, by Araki-James-Thomas [2]. Moreover Hubbuck[13] proved that if a connected finite CW-complex Y is a homotopy commutative H-space, then Y has the homotopy type of a torus.

In this paper, we are concerned with the homotopy commutativity of the loop space ΩX of a connected, simply connected finite CW-complex X. It is easy to see that

(i) If X itself is an H-space, then ΩX is homotopy commutative.

But the converse is not true for the complex projective 3-space $CP(3)$. In fact, Stasheff [21; Th.1.18] proved that $\Omega CP(3)$ is homotopy commutative; but $CP(3)$ is not an H-space which is seen by Borel's theorem on the cohomology ring of an H-space.

We note also that X is an H-space if and only if ΩX is strongly homotopy commutative in the sense of Sugawara [23].

Now the purpose of this paper is to prove the following

THEOREM 1. *Let X be the suspension $X = \Sigma A$ of a connected finite CW-*