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The maximal codegree of the quaternionic projective spaces

Dedicated to Professor Akio Hattori on his sixtieth birthday

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§1. Introduction

For a k-dimensional oriented vector bundle α over a connected finite CW-complex X, the codegree $cd(X^\alpha)$ of the Thom space X^α is defined by

$$cd(X^\alpha) = |\mathrm{Coker}\,[h : \pi_s^k(X^\alpha) \to H^k(X^\alpha; Z)]|\,,$$

the order of the cokernel of the stable Hurewicz homomorphism h of the stable cohomotopy group to the integral cohomology group. We study this codegree by restricting our attention to

$$cd_2(X^\alpha) = \nu_2(cd(X^\alpha))\,,$$

the exponent of 2 in the prime power decomposition of $cd(X^\alpha)$. The cohomology groups are always assumed to be reduced.

Let kO (resp. $kSpin$) be the -1 (resp. 3) connective cover of the KO-spectrum KO. Then the spectrum j is defined to represent the fiber of

$$\psi : kO^*(\ )_{(2)} \to kSpin^*(\ )_{(2)} \quad (G_{(2)} \text{ is the localization of } G \text{ at } 2)$$

which is a unique lift of the stable Adams operation $\psi^3 - 1 : KO^*(\ )_{(2)} \to KO^*(\ )_{(2)}$ (cf. [16], [6], [17]); and we have the Hurewicz homomorphism

$$h_j : j^k(X^\alpha) \to H^k(X^\alpha; Z_{(2)})$$

which factors $h : \pi_s^k(X^\alpha) \to H^k(X^\alpha; Z_{(2)})$. Thus we have the j-codegree

$$cd_2^j(X^\alpha) = \nu_2(|\mathrm{Coker}\,(h_j)|) \qquad \text{with} \qquad cd_2^j(X^\alpha) \leqq cd_2(X^\alpha)\,,$$

which has another description being available for calculations (see Corollary 2.7).

Now, M. C. Crabb and K. Knapp introduced the notion of the maximal codegree given as follows:

THEOREM A (Crabb-Knapp[7]). *For any integer n, put*

(1.1) $\quad m_2(n) = [n/2]$ *if* $\quad n \equiv 0, 1, 2, 6, 7 \bmod 8, = [n/2] + 1 \quad$ *otherwise*.