

A distortion theorem for conformal mappings with an application to subharmonic functions

Dedicated to Professor T. Fujiwara on his 60th birthday

Makoto MASUMOTO

(Received June 27, 1989)

1. Introduction

Let f be a conformal mapping of a domain D_0 of \mathbf{C} bounded by a finitely many analytic curves. Our first purpose of this paper is to establish the following relation between $|f'(z)|$ and the Poincaré metric $\lambda_{D_0}(z)$ for D_0 .

THEOREM I. *If $f(D_0)$ satisfies an exterior θ -wedge condition, then there exists $m > 0$ such that*

$$\frac{1}{|f'(z)|} \leq m\lambda_{D_0}(z)^{1-\theta}$$

for $z \in D_0$.

Here, a finitely connected domain D is said to satisfy an exterior θ -wedge condition if it is bounded and there exist $\rho > 0$ and $\theta \in (0, 1)$ such that, for every $\omega \in \partial D$, a closed sector of radius ρ and opening $\pi\theta$ with vertex at ω lies in $\mathbf{C} - D$.

Recently, N. Suzuki has obtained the following theorem:

THEOREM (Suzuki [5, Theorem 2]). *Let D be a bounded $C^{1,1}$ -domain of \mathbf{C} , and denote by $\delta_D(z)$ the distance between $z \in D$ and ∂D . Set $\alpha(p) = 1 + \max\{1 - p, 0\}$ for $0 < p < \infty$. If a nonnegative subharmonic function s on D satisfies*

$$\iint_D \delta_D(z)^{-\alpha(p)} s(z)^p dx dy < +\infty, \quad z = x + iy,$$

then s must vanish identically.

We apply Theorem I to generalize Suzuki's theorem:

THEOREM II. *Suppose that D satisfies an exterior θ -wedge condition. Set $\beta(p, \theta) = 2 - \min\{1, p\}/(2 - \theta)$. If a nonnegative subharmonic function s on D satisfies*