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Linearized oscillations for equations with positive and negative coefficients

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1. Introduction

Recently a linearized oscillation theory has been developed in [6]–[9] for nonlinear delay differential equations which in some sense parallels the so called linearized stability theory for differential equations. Roughly speaking, it has been shown that, under appropriate hypotheses, certain nonlinear differential equations have the same oscillatory behavior as an associated linear equation with constant coefficients.

Our aim in this paper is to present a linearized oscillation result for the neutral differential equation with positive and negative coefficients

$$\frac{d}{dt}[x(t) - P(t)G(x(t - \tau))] + Q_1(t)H_1(x(t - \sigma_1)) - Q_2(t)H_2(x(t - \sigma_2)) = 0 \tag{1}$$

where we will assume that there are constants P_0, p_0, q_1, q_2 and M such that the following hypotheses are satisfied:

$$P,\ Q_1,\ Q_2 \in C[[t_0, \infty), R^+]\,, \qquad G,\ H_1,\ H_2 \in C[R, R]\,, \tag{2}$$

$$\tau \in (0, \infty)\,, \quad \sigma_1, \sigma_2 \in [0, \infty)\,, \qquad \sigma_1 \geqq \sigma_2\,, \tag{3}$$

$$\limsup_{t\to\infty} P(t) = P_0 \in (0, 1)\,, \qquad \liminf_{t\to\infty} P(t) = p_0 \in (0, 1)\,, \tag{4}$$

$$\liminf_{t\to\infty} Q_1(t) = q_1\,, \qquad \limsup_{t\to\infty} Q_2(t) = q_2\,, \tag{5}$$

$$0 \leqq \frac{G(u)}{u} \leqq 1 \qquad \text{for } u \neq 0\,, \qquad \lim_{u\to 0} \frac{G(u)}{u} = 1\,, \tag{6}$$

$$\frac{H_1(u)}{H_2(u)} \geqq 1 \qquad \text{and} \qquad 0 < \frac{H_2(u)}{u} \leqq M \qquad \text{for } u \neq 0, \qquad \lim_{u\to 0} \frac{H_2(u)}{u} = 1 \tag{7}$$

and

$$1 - P_0 - Mq_2(\sigma_1 - \sigma_2) > 0\,. \tag{8}$$

With Eq.(1) we associate the linear equation with constant coefficients

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