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Sobolev norms of radially symmetric oscillatory solutions for superlinear elliptic equations

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1. Introduction

In this paper we consider the radially symmetric solutions for the semilinear elliptic equation

$$\Delta u + g(u) = 0\,, \qquad x \in \Omega\,, \tag{1.1}$$

$$u = 0\,, \qquad x \in \partial\Omega\,, \tag{1.2}$$

where $\Omega \equiv \{x \in \mathbf{R}^n : |x| < 1\}$, $n \geq 2$. The nonlinear function $g(s)$ is supposed to be a continuous function with the following properties:

$$(\mathrm{g}_1) \qquad \begin{aligned} &g(0) = 0\,, \qquad \lim_{|s|\to\infty} g(s)/s = \infty \qquad \text{and} \\ &g'(0) = \lim_{s\to 0} g(s)/s \quad \text{exists}\,. \end{aligned}$$

Hence (1.1) may be called a superlinear elliptic equation. For radially symmetric solutions $u = u(t)$, $t = |x|$, Equation (1.1) is converted to the boundary value problem for the second order ordinary differential equation

$$u'' + \frac{n-1}{t}u' + g(u) = 0\,, \qquad t \in (0, 1)\,, \tag{1.3}$$

$$u'(0) = 0\,, \qquad u(1) = 0\,. \tag{1.4}$$

Equation (1.3) can be written as

$$(t^{n-1}u')' + t^{n-1}g(u) = 0\,, \qquad t \in (0, 1)\,, \tag{1.3$'$}$$

so that we can treat the problem (1.3)′–(1.4) as a singular boundary value problem for a nonlinear Sturm-Liouville equation.

Under some growth conditions on $g(s)$, Ambrosetti and Rabinowitz established in [1] that the semilinear problem (1.1)–(1.2) formulated in an arbitrary bounded domain Ω possesses infinitely many solutions and moreover $H_0^1(\Omega)$ norms of solutions assume arbitrarily large values. Related problems are treated in [2, 8, 13, 18, 19, 20]. In the case where Ω is the unit ball, the existence of infinitely many radially symmetric solutions has been investigated by Castro-Kurepa [4] and Struwe [21] (see also [12] in the case of the whole space