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Existence theorems for Monge-Ampère equations in R^N

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1. Introduction

Our aim is to establish the existence of positive radial entire solutions $u(x)$ of nonlinear partial differential equations of the Monge-Ampère type

$$\det(D^2u) = \alpha\Delta u + \lambda f(|x|, u, |Du|), \qquad x \in \boldsymbol{R}^N, \qquad N \geq 3 \tag{1}$$

which grow like constant multiples of $|x|^2$ as $|x| \to \infty$, where $\alpha > 0$ and $\lambda \in \boldsymbol{R}$ are constants and $f \in C(\boldsymbol{D}, \boldsymbol{R})$, $\boldsymbol{D} = \bar{\boldsymbol{R}}_+ \times \boldsymbol{R}_+ \times \bar{\boldsymbol{R}}_+$, $\boldsymbol{R}_+ = (0, \infty)$, $\bar{\boldsymbol{R}}_+ = [0, \infty)$. Detailed hypotheses on f are listed in §2. Under modified conditions we also prove (Theorem 3) the existence of radial entire solutions of (1) which are positive in some neighborhood of infinity.

As usual, $|x|$ denotes the Euclidean length of a point $x = (x_1, \dots, x_N)$ in $\boldsymbol{R}^N$, $D_i = \partial/\partial x_i$, $D_{ij} = D_iD_j$ for $i, j = 1, \dots, N$, $Du = (D_1u, \dots, D_Nu)$, $\Delta = \Sigma_{i=1}^N D_{ii}$, and D^2u is the Hessian matrix $(D_{ij}u)$.

An *entire solution* of (1) is defined to be a function $u \in C^2(\boldsymbol{R}^N)$ satisfying (1) at every point $x \in \boldsymbol{R}^N$. We seek radially symmetric entire solutions $u(x) = y(t)$, $t = |x|$, of (1) such that

$$0 < \liminf_{t\to\infty} t^{-2}y(t), \qquad \limsup_{t\to\infty} t^{-2}y(t) < \infty. \tag{2}$$

In particular our results apply to the following special cases of (1):

$$\det(D^2u) = \alpha\Delta u + \lambda p(|x|)u^\gamma, \qquad x \in \boldsymbol{R}^N; \tag{3}$$

$$\det(D^2u) = \alpha\Delta u + \lambda p(|x|)e^u, \qquad x \in \boldsymbol{R}^N, \tag{4}$$

where γ is a positive constant and $p \in C(\bar{\boldsymbol{R}}_+, \boldsymbol{R})$. If $p(t) = 0(t^{-2\gamma})$ as $t \to \infty$, Theorem 1 implies that (3) has an infinitude of positive radial entire solutions $u(x) = y(|x|)$ satisfying (2), for all sufficiently small $|\lambda|$. If in addition $\gamma < N$ and $p(t) \geq 0$ on $\bar{\boldsymbol{R}}_+$, Theorem 2 shows that (3) has positive radial entire solutions satisfying (2) for all $\lambda \geq 0$.

If $p(t) = 0[\exp(-2\alpha_N t^2)]$ as $t \to \infty$, where

$$\alpha_N = (\alpha N)^{1/(N-1)}, \qquad N \geq 3, \tag{5}$$

Theorem 1 implies that (4) has an infinitude of positive radial entire solutions $u(x) = y(|x|)$ satisfying (2) for sufficiently small $|\lambda|$. Theorem 3 establishes, for