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Invariant measures and entropies of random dynamical systems and the variational principle for random Bernoulli shifts

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§ 1. Introduction

We consider a dynamical system in a compact metric space (M, d) in which continuous maps operated on points are successively chosen randomly according to a fixed probability law. Such random dynamical systems were studied, for example, by [3], [4], [7] and [8]. More precisely, let Φ be a set of maps with a measurable structure $\mathscr{F}$ and $\{\varphi_n(\omega): n \in \mathbf{N}\}$ be $(\Phi, \mathscr{F})$-valued stochastic process on a probability space $(\Omega, \mathfrak{F}, \mathbf{P})$. The corresponding trajectories on M are $\{\varphi_n(\omega) \circ \cdots \circ \varphi_1(\omega)x: n \in \mathbf{N}\}$, $x \in M$, for $\omega \in \Omega$. Here the underlying probability space $(\Omega, \mathfrak{F}, \mathbf{P})$ can be taken to be $(\Phi^{\mathbf{N}}, \mathscr{F}^{\mathbf{N}}, P)$ for some P. To avoid the dependence of the law for the choice of maps on time, we impose on P the stationality (i.e. the shift invariance). We also assume that P is ergodic for simplicity.

Define a map τ on $M \times \Phi^{\mathbf{N}}$ by $\tau(x, \tilde{\varphi}) = (\varphi_1 x, \sigma\tilde{\varphi})$, $x \in M$, $\tilde{\varphi} = (\varphi_1, \varphi_2, \dots) \in \Phi^{\mathbf{N}}$, where σ is the shift transformation on $\Phi^{\mathbf{N}}$. This map τ is called the skew product transformation. In most articles, a probability measure of the following type was considered as an invariant measure of τ: $Q = \mu \times P$, where $P = \rho^{\mathbf{N}}$, ρ is a probability measure on $(\Phi, \mathscr{F})$ and μ is a stationary (i.e. invariant) probability measure of the transition probability $P(x, B) = \int_\Phi 1_B(\varphi x) d\rho(\varphi)$. Tsujii [10] treated a slightly different measure in connection with the theory of random fractals. He gave a τ-invariant measure which has the non-trivial decomposition with respect to the partition $\{M \times \{\tilde{\varphi}\}: \tilde{\varphi} \in \Phi^{\mathbf{N}}\}$. Even in his system P turns out to be of the type $\rho_\gamma^{\mathbf{N}}$ for some probability measure ρ_γ on Φ.

In this paper we consider the random dynamical sysytems in more general situation. We assume only that an ergodic shift invariant probability measure P on $(\Phi^{\mathbf{N}}, \mathscr{F}^{\mathbf{N}})$ is given and we are concerned with a τ-invariant Q on $(M \times \Phi^{\mathbf{N}}, \mathscr{B}_M \times \mathscr{F}^{\mathbf{N}})$ which is required to satisfy only the condition $Q(M \times F) = P(F)$, $F \in \tilde{\mathscr{F}}$. In § 2 we prove the existence of such a τ-invariant probability measure Q. When maps are homeomorphisms, as a natural extention of $\Phi^{\mathbf{N}}$, we can take the underlying probability space to be $(\Phi^{\mathbf{Z}}, \mathscr{F}^{\mathbf{Z}}, P)$ for a given ergodic shift