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Martin boundary of a harmonic space with adjoint structure and its applications

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Introduction

Consider the parabolic operator

$$Lu(x, t) = a(x)\frac{\partial u}{\partial t} - \Delta_x u + \langle \boldsymbol{b}(x, t), \nabla_x u\rangle + c(x, t)u$$

and its adjoint operator

$$L^*u(x, t) = -a(x)\frac{\partial u}{\partial t} - \Delta_x u - \langle \boldsymbol{b}(x, t), \nabla_x u\rangle + c^*(x, t)u$$

with sufficiently smooth coefficients $a > 0$, $\boldsymbol{b}$ ($\boldsymbol{R}^n$-valued), c and $c^* = c - \nabla_x \boldsymbol{b}$ on a domain D in $\boldsymbol{R}^n \times \boldsymbol{R}$. If we write $\hat{L} = (L + L^*)/2$, then, noting that $c = L1$ and $c^* = L^*1$, we have

$$\Delta_x u = -\hat{L}u + u\hat{L}1.$$

Therefore, if the "lateral" boundary $\partial_s D$ of D is sufficiently regular, then for f, $g \in C^2(\bar{D})$ such that g vanishes on $\partial_s D$, Green's formula implies

$$\int_D \langle \nabla_x f, \nabla_x g\rangle\, dxdt + \int_D fg\,\hat{L}1\, dxdt = \int_D g\,\hat{L}f\, dxdt, \tag{0.1}$$

provided that all the integrals exist.

The purpose of the present paper is to establish a formula corresponding to (0.1) on a harmonic space $(X, \mathscr{H})$ with an adjoint structure $\mathscr{H}^*$, as an application of the theory of Martin boundary of X with respect to the structures $\mathscr{H}$ and $\mathscr{H}^*$.

In §2—§6, we develop a theory of Martin boundary of such a harmonic space $(X, \mathscr{H})$. Theories of Martin boundary of general harmonic spaces have been discussed to some extent by M. Sieveking [8], K. Janssen [3] and C. Constantinescu–A. Cornea [1; Chapter 11]; and some results in §2—§6 of the present paper can be obtained from these general theories. However, in order to obtain some properties which we need in establishing the above mentioned formula, we rather follow the classical approaches by Martin–Brelot–Naim and