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Asymptotic periodicity of densities and ergodic properties for nonsingular systems

Tomoki INOUE and Hiroshi ISHITANI

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§ 0. Introduction

Each one dimensional piecewise smooth expanding transformation T on a finite interval has the following ergodic property (A), which is the result of Li and Yorke [11] and Wagner [15], (see also Morita [13]). In one dimensional case m denotes the Lebesgue measure.

(A) *There exists a sequence of m-absolutely continuous T-invariant probability measures* $\{\mu_1, \cdots, \mu_l\}$ *with* $L_i := \operatorname{supp} \mu_i$ *for* $i = 1, \cdots, l$, *which has the following properties.*

(1) $\mu_i(L_i) = 1$ *for* $i = 1, \cdots, l$.

(2) (T, μ_i) *is ergodic for* $i = 1, \cdots, l$.

(3) $m(L_i \cap L_j) = 0$ *if* $i \neq j$.

(4) $T^{-1}(L_i) \supset L_i$ $m-a.e.$ *for* $i = 1, \cdots, l$.

(5) *If* η *is an m-absolutely continuous T-invariant probability measure,* η *can be written as a convex combination of* μ_i*'s.*

(6) *Put* $C = \bigcup_{n=0}^{\infty} \{x;\ T^n(x) \notin \bigcup_{i=1}^{l} L_i\}$, *then* $m(C) = 0$.

(7) *For* $i = 1, \cdots, l$, *there exists a collection of sets* $L_{i1}, \cdots, L_{ir(i)}$ *with the following properties*:

(*a*) $L_i = \bigcup_{j=1}^{r(i)} L_{ij}$.

(*b*) $m(L_{ij} \cap L_{ik}) = 0$ *if* $j \neq k$.

(*c*) $T^{-1}(L_{i,j+1}) \supset L_{ij}$ $m-a.e.$ *for* $j = 1, \cdots, r(i) - 1$, and $T^{-1}(L_{i1}) \supset L_{i,r(i)}$ $m-a.e.$

(*d*) $(T^{r(i)}, \mu_{ij})$ *is exact, where* $\mu_{ij} = r(i) \cdot \mu_i|_{L_{ij}}$.

On the other hand, Lasota, Li and Yorke [8] pointed out that the behavior of the Frobenius-Perron operator P associated with T is asymptotically periodic. Namely it has the following property (B).

(B) *There exists a sequence of densities* $g_1, \cdots, g_r$ *and a sequence of bounded linear functionals* $\lambda_1, \cdots, \lambda_r$ *such that*

$$\lim_{n \to \infty} \| P^n(f - \textstyle\sum_{i=1}^{r} \lambda_i(f) g_i) \|_{L^1(m)} = 0 \qquad \text{for } f \in L^1(m),$$

the densities $\{g_i\}$ *have mutually disjoint supports* ($g_i g_j = 0$ *for* $i \neq j$) *and*

$$P g_i = g_{\alpha(i)}$$