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A spectrum whose BP_*-homology is $(BP_*/I_5)[t_1]$

Katsumi SHIMOMURA

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§1. Introduction

For each prime p, we have the Brown-Peterson spectrum BP whose coefficient is the polynomial ring $BP_* = \mathbf{Z}_{(p)}[v_1, v_2, \cdots]$ over Hazewinkel's generators v_i with $|v_i| = 2p^i - 2$. This has the invariant prime ideals $I_n = (p, v_1, \cdots, v_{n-1})$ for $n \geq -1$, where $I_{-1} = (0)$ and $I_0 = (p)$. Then the Toda-Smith spectrum $V(n)$ is the finite ring spectrum characterized by

$$BP_* V(n) = BP_*/I_{n+1}$$

for $n \geq -1$. Once we know the existence of this spectrum, we can construct a family of nontrivial elements of the homotopy groups $\pi_* S$ of the sphere spectrum S, which are known as the Greek letter elements. The existence of the spectrum $V(n)$ is known only for $n < 4$. In this case $V(n)$ exists if and only if the prime p is greater than $2n$. It seems that $V(4)$ exists for a large prime p, but still now we have no way to prove it. We so consider a similar spectrum $W_k(n)$ defined by

$$BP_* W_k(n) = (BP_*/I_{n+1})[t_1, \cdots, t_k]$$

as a BP_*BP-comodule subalgebra of $BP_*BP/I_{n+1} = (BP_*/I_{n+1})[t_1, t_2, \cdots]$. Then $V(4) = W_0(4)$. If $W_k(n)$ does not exist for some k, neither does $V(n)$. However by computing obstructions we obtain the existence of $W_k(4)$ for $k > 1$ at a prime $p > 7$ in [6], and in this paper we prove the following

THEOREM. *Let p be a prime number greater than 7. Then $W_1(4)$ exists.*

In §2 we recall Ravenel's ring spectra $T(k)$ and show the following

PROPOSITION. *Let p be any prime and k and n non-negative integers with $k \geq n$. Then there exists a $T(k)$-module spectrum $W_k(n)$.*

In §§3–4 we compute the differentials of the Adams-Novikov spectral sequence for the spectrum $W_1(3)$ and show the above theorem.

§2. $W_k(n)$

Let p denote an odd prime number and S be the sphere spectrum. The