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Infinitely many radially symmetric solutions of certain semilinear elliptic equations

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1. Introduction

In this paper we consider radially symmetric solutions to the semilinear elliptic problem

$$\Delta u + f(u) = 0, \qquad x \in \boldsymbol{R}^n, \tag{1.1}$$

$$\lim_{|x| \to \infty} u(x) = 0, \tag{1.2}$$

where $n \geq 2$ and $f(u)$ is locally Lipschitz continuous. The problem of finding radially symmetric solutions $u = u(t)$, $t = |x|$, of equation (1.1) subject to condition (1.2) is converted to the singular boundary value problem for the ordinary differential equation

$$u'' + \frac{n-1}{t} u' + f(u) = 0, \quad t > 0, \tag{1.3}$$

$$u'(0) = 0, \tag{1.4}$$

$$\lim_{t \to \infty} u(t) = 0. \tag{1.5}$$

Under the condition that

$$sf(s) < 0 \quad \text{for} \quad |s| > 0 \quad \text{sufficiently small,} \tag{1.6}$$

the existence of infinitely many solutions to the problem (1.3)–(1.5) has been obtained by several authors. Assumption (1.6) arises from the study of standing wave solutions of the nonlinear Klein-Gordon or Schrödinger equations (see the references [1], [2], [10]). Berestycki and Lions [1], Berger [2] and Strauss [10] obtained the existence results of infinitely many solutions by means of variational methods. They treated this problem in the case where the function $f(s)$ is odd, $f'(0) < 0$ and satisfies some growth conditions. On the other hand, using a dynamical system approach, Jones and Küpper [5] have proved that for any integer $k \geq 0$ there exists a solution of (1.3)–(1.5) having exactly k zeros in the interval $[0, \infty)$. Under the assumption (1.6) which is weaker than the condition $f'(0) < 0$, McLeod, Troy and Weissler [7] have