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An oscillation criterion for Sturm-Liouville equations with Besicovitch almost-periodic coefficients

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Let $\boldsymbol{R}$ denote the real line. The class $\Omega \subset L^1_{loc}(\boldsymbol{R})$ of Besicovitch almost-periodic functions is the closure of the set of all finite trigonometric polynomials with the Besicovitch seminorm $\|\cdot\|_B$:

$$\|p\|_B := \limsup_{t\to\infty} \frac{1}{2t}\int_{-t}^{t} |p(s)|\,ds,$$

where $p\in\Omega$. The mean value, $M\{p\}$, of $p\in\Omega$ always exists, is finite, and is uniform with respect to α for $\alpha\in\boldsymbol{R}$, where

$$M\{p\} := \lim_{t\to\infty} \frac{1}{t}\int_{t_0}^{t} p(s+\alpha)\,ds,$$

for some $t_0 \geq 0$ (see [1] and [2] for details).

Consider the second order nonlinear differential equation

$$\text{(E)} \qquad x''(t) - \lambda p(t) f(x(t)) = 0,$$

where $p\in\Omega$, $f\in C(\boldsymbol{R};\boldsymbol{R})$ and $\lambda\in\boldsymbol{R}-\{0\}$.

Equation (E) is oscillatory at $+\infty$ and $-\infty$ if every continuable solution of (E) has an infinity of zeros clustering only at $+\infty$ and $-\infty$, respectively.

Recently, A. Dzurnak and A. B. Mingarelli [3] proved the following very interesting result by using Levin's comparsion theorem [5].

THEOREM A. *Let $p\in\Omega$ and $M\{|p|\}>0$. If f is the identity mapping, then* (E) *is oscillatory at $+\infty$ and $-\infty$ for every $\lambda\in\boldsymbol{R}-\{0\}$ if and only if $M\{p\}=0$.*

The purpose of this note is to extend Theorem A to the nonlinear case by using the following nonlinear version of Levin's comparsion theorem which is due to Yeh [8].

THEOREM B. *Let*

(C_1) *$f\in C^1(\boldsymbol{R}-\{0\})$ such that $xf(x)>0$ and $f'(x)>0$ for all $x\neq 0$,*

(C_2) *f' is decreasing on $(0,\infty)$ and increasing on $(-\infty,0)$,*