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Markov-self-similar sets

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1. Introduction

A theory of non-random self-similar sets has been developed by Moran [11] and Hutchinson [9]. Lately Mauldin-Williams [10], Falconer [5] and Graf [7] investigated random self-similar sets. In this paper we introduce a new concept of *Markov-self-similarity* and investigate deterministic and random Markov-self-similar sets. Takahashi [12] introduced a concept of multi-similarity which is essentially the same concept as Markov-self-semilarity. Markov-self-similarity is a natural extension of self-similarity and Markov-self-similar sets appear as the limit sets of cellular automata [12, 15]. Cellular automata are used to model problems in crystal growth and diffusion and other problems of self-organization. Therefore the patterns appeared in these fields are expected to be Markov-self-similar. On the other hand some Markov-self-similar sets can be constructed as recurrent sets defined by Dekking [3]. (See also Bedford [1, 2].)

A Markov-self-similar set is constructed as follows. First we prepare an N-tuple $(S_{01}, \dots, S_{0N})$ of contraction similarities of $\mathbf{R}^d$ which are initial contractions and used only in the first step. Let F be a non-empty compact subset of $\mathbf{R}^d$, and set

$$A_1 = \bigcup_{k=1}^{N} S_{0k}(F).$$

Next we fix a family of N N-tuples $\{(S_{k1}, \dots, S_{kN})\}_{k=1}^{N}$ of contraction similarities of $\mathbf{R}^d$ which are fundamental contractions and used in the following process repeatedly. We assume that above N N-tuples satisfy the irreducibility condition and the open set condition. (See Section 2.) Set

$$A_2 = \bigcup_{k=1}^{N} S_{0k}(\bigcup_{i=1}^{N} S_{ki}(F)).$$

Note that the contractions S_{ki} are selected depending on the index k of S_{0k}. Set

$$A_3 = \bigcup_{k=1}^{N} S_{0k}(\bigcup_{i=1}^{N} S_{ki}(\bigcup_{j=1}^{N} S_{ij}(F))).$$

We continue this process. Let $K = \lim_{n\to\infty} A_n$ where the limit is taken with respect to the Hausdorff metric. The set K has a Markovian shape structure which is not possessed by a self-similar set constructed in Hutchinson [9].