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3-valued problem and reduction of some integer programming problems

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1. Introduction

The so-called *integer linear programming problem* is described as follows; Given integers

(1.1) $$a_{ij},\ b_i \text{ and } c_j,\ 1 \leqq i \leqq m,\ 1 \leqq j \leqq n,$$

find non-negative integers $x_1, \dots, x_n$ such that $\sum_{j=1}^n c_j x_j$ takes the maximum value under the constraints

(1.2) $$\sum_{j=1}^n a_{ij} x_j \leqq b_i,\ i = 1, \dots, m.$$

Various methods for solving this problem have been discussed in [3, 5, 11, 18, 20, 22, 24]. Especially, useful methods are exploited in [9, 10, 13, 14, 17, 35] for 2-valued problems in which a_{ij}, x_j are supposed to belong to $\{0, 1\}$. The 2-valued problems can be applied to various problems concerning graphs, networks, and so on.

In this paper we are concerned with the following four types of integer problems:

Integer Selection Problem, or shortly ISP: Let n, k, $m_1, \dots, m_k$ be positive integers and let a_{ij}^r, b_i^r, c_j, $1 \leqq i \leqq m_r$, $1 \leqq r \leqq k$, $1 \leqq j \leqq n$ and z be given integers. Find an integer r with $1 \leqq r \leqq k$ and non-negative integers $x_1, \dots, x_n$ satisfying

$$\sum_{j=1}^n c_j x_j = z \text{ and } \sum_{j=1}^n a_{ij}^r x_j \leqq b_i^r \text{ for } 1 \leqq i \leqq m_r.$$

3-valued Problem: Given integers $a_{ij} \in \{-1, 0, 1\}$ and b_i stated in (1.1), find $x_1, \dots, x_n \in \{0, 1\}$ satisfying (1.2).

Indeterminate Coefficient Problem, or shortly IDCP: Let m, n, p and q be positive integers, a_{ij}, b_i integers given in (1.1), and let g_{si}, $d_{jt} \in \{-1, 0, 1\}$ and ℓ_{st}, $1 \leqq s \leqq p$, $1 \leqq t \leqq q$ be given integers. Find non-negative integers x_j and $y_{ij} \in \{0, 1\}$ satisfying

$$\sum_{j=1}^n a_{ij} y_{ij} x_j \leqq b_i,\ i = 1, \dots, m \text{ and}$$

$$\sum_{i=1}^m \sum_{j=1}^n g_{si} y_{ij} d_{jt} \leqq \ell_{st},\ s = 1, \dots, p,\ t = 1, \dots, q.$$