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Tests for random-effects covariance structures in the growth curve model with covariates

Takahisa YOKOYAMA and Yasunori FUJIKOSHI

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1. Introduction

Suppose that we obtain serial measurements for each of N individuals on each of p occasions, yielding an $N \times p$ data matrix of observations X. The growth curve model for the observation matrix X of Potthoff and Roy [8] can be written as

$$X = A\Xi B + U, \tag{1.1}$$

where A is an $N \times k$ design matrix across individuals, Ξ is a $k \times q$ matrix of unknown parameters, B is a $q \times p$ design matrix within individuals, and U is an $N \times p$ unobservable matrix of random errors. It is assumed that A and B have ranks k and q, respectively, and the rows of U are independently and identically distributed as $N_p(\mathbf{0}, \Sigma)$, where Σ is an unknown $p \times p$ positive definite matrix. For an extensive survey of the literature on the model (1.1), see, e.g., Timm [11], Geisser [4] and Woolson [12]. In the model (1.1), suppose that we can use the observations of r covariates for the N individuals. Let Z be the $N \times r$ observation matrix of r covariates. Then the model (1.1) can be extended as

$$X = A\Xi B + Z\Theta + U, \tag{1.2}$$

where Θ is an $r \times p$ matrix of unknown parameters. It is assumed that Z is fixed and $\operatorname{rank}[A, Z] = k + r \leq N - p$. This type of models has been considered in Chinchilli and Elswick [3].

When there is no theoretical or empirical basis for assuming special covariance structures, we need to assume that Σ is an arbitrary positive definite covariance matrix. However, when p is large relative to N, more parsimonious covariance structures are required. Rao [9], [10] introduced a natural candidate for such parsimonious covariance structures, based on random-effects models. As a generalization of his idea we consider a family of covariance structures (see Lange and Laird [7])

$$\Sigma = B_c' \Delta_c B_c + \sigma_c^2 I_p, \qquad 0 \leq c \leq q, \tag{1.3}$$