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The projection method for accelerated life test model in bivariate exponential distributions

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1. Introduction

Marshal and Olkin [6] have introduced the bivariate exponential distribution ($BVED$) with survival distribution

$$P(X \geq x, Y \geq y) = \bar{F}(x, y) = \exp\{-\lambda_1 x - \lambda_2 y - \lambda_0 \max(x, y)\},$$

where λ_0, λ_1 and λ_2 are parameters. Let U_0, U_1 and U_2 be independently distributed from univariate exponential distributions with failure rates λ_0, λ_1 and λ_2, respectively. Then (X, Y) can be written as $X = \min(U_1, U_0)$ and $Y = \min(U_2, U_0)$. Thus X and Y come from the shock times U_1 and U_2, respectively and are simultaneously governed by the fatal shock causing at the time U_0. For a sample from a $BVED$ the likelihood equation system for parameters λ_0, λ_1 and λ_2 is a simple algebraic one but the solution is intractable, cf. Arnold [1]. So one has to use some iteration method, e.g., the Fisher scoring method to seek the numerical value of the maximum likelihood estimator (MLE). Arnold introduced the unbiased estimator of the simple form, which has fairly less relative efficiency to the MLE on a part of the parameter space. Proschan and Sullo [7] proposed the intuitive estimator with high relative efficiency over all the space, which is not fully efficient. We give an efficient estimator by the projection method, see Eguchi [4] for formal derivation and several applications of the method. The estimator form is less simple than other non-iterative estimators but the construction is necessary as the first stage in the following further analysis.

Ebrahimi [3] has considered a bivariate accelerated life test model, see also Basu and Ebrahimi [2] for nonparametric approaches and Mann, Shafer and Singpuwala [5] for general notion as power rule model.

The main purpose of this paper is to establish the projection method of testing and estimation for the accelerated life test model. Let (X_j, Y_j) be independently distributed from a $BVED$ for $j = 1, \ldots, J$ with parameters λ_{0j}, λ_{1j}, λ_{2j} which satisfy

$$\lambda_{ij} = C_i V_j^P \qquad (i = 0, 1, 2) \tag{1.1}$$

Here V_j is a controllable variable which denotes the stress level at the j-th