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Classification of weighing matrices of small orders

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Summary

The classification problem of weighing matrices of orders not exceeding 14 has been completed by Chan et al. [2] and Ohmori [17, 18]. In this paper, we first consider a construction problem of weighing matrices of order $8a-2$ and weight $4a$ for $a \geq 2$. A general solution for the intersection pattern condition, which is necessary to construct such weighing matrices, is given. Furthermore, the complete classification of weighing matrices for the case $a = 2$ is made.

1. Introduction

A weighing matrix W of order n and weight k is an $n \times n$ matrix with elements $+1$, -1 and 0 such that $WW^t = kI_n$, $k \leq n$, where I_n is the identity matrix of order n and W^t denotes the transpose of W. We refer to such a matrix as a $W(n, k)$. A $W(n, n)$ is called a Hadamard matrix of order n. It is known that the order of a Hadamard matrix is 2 or a multiple of 4. In fact, the concept of weighing matrices was introduced by Taussky [24] as a generalization of Hadamard matrices. However, in the area of design theory, weighing matrices appear naturally as the "coeffi ent" matrices of an orthogonal design (see Geramita and Seberry [4]) and s applications for weighing designs (for example, see Chakrabarti [1], Federer [3], Raghavarao [22]). Furthermore, weighing matrices have been studied in order to find optimal solutions to the so-called weighing design problem of weighing objects whose weights are small relative to the weights of moving parts of the balance being used. It was shown by Raghavarao [21, 22] that if the variance of the errors in the weights obtained by individual weighing is σ^2 in the usual weighing design set up, then using a $W(n, k)$ as a design of an experiment to weigh n objects will give the variance σ^2/k. Indeed, in the class of all such weighing designs for $n \equiv 0 \pmod 4$, a Hadamard matrix is optimal. Furthermore, in the class of all weighing designs for $n \equiv 2 \pmod 4$, a symmetric conference matrix (that is a kind of $W(n, n-1)$) is optimal. Weighing matrices also have applications in the area of coding theory. A linear code is an l-dimensional subspace of the m-dimensional space over Galois field $GF(q)$. The