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# Asymptotic behavior of a biological model with time delays

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## 1. Introduction

In this paper we study the scalar delay differential equation

$$x'(t) = \sum_{i=1}^{n} b_i x(t - r_i)(1 - ax(t)) - cx(t) \tag{1.1}$$

where $r_i > 0$, $b_i > 0$ $(i = 1, 2, \cdots, n)$, $a > 0$, $b = \sum_{i=1}^{n} b_i > 0$, $c \geq 0$. If we take $n = 1$, $a = 1$ and $c > 0$, this equation becomes the epidemic model given by K. L. Cooke in [4, 5], that is

$$x'(t) = bx(t - \tau)(1 - x(t)) - cx(t).$$

K. L. Cooke made the following assumptions on his model (in this paper we also use them.):
(a) The infection is transmitted to man by a vector, such as a mosquito. Susceptible persons receive the infection from infectious vectors, and susceptible vectors receive the infection from infectious persons.
(b) The human population in the community under consideration is fixed, hence we are interested in the solution $x(t)$ of (1.1) which obeys $0 \leq x(t) \leq 1$. The infection in humans does not result in death or isolation.
(c) When a susceptible vector is infected by a person, there is a fixed time during which the infectious agent develops in the vector. At the end of this time the vector can infect a susceptible human.
(d) Infected humans have a constant recovery rate c. Note that the time during which the infectious agent develops in vectors of different species may be different, so the following model which is a special case of (1.1) may be more reasonable

$$x'(t) = \sum_{i=1}^{n} b_i x(t - r_i)(1 - x(t)) - cx(t) \qquad t \geq 0 \tag{1.2}$$

where $b_i > 0$, $r_i > 0$ $(i = 1, 2, \cdots, n)$, $c > 0$ are constants. On the other hand, when $c = 0$ and $n = 1$, (1.1) is the Logistic model given by K. Gopalsamy in [7].