HIROSHIMA MATH. J.
22 (1992), 103–113

Cofine boundary behaviour of temperatures

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(Received November 5, 1990)

1. Introduction, notation and terminology

The Dirichlet problem for the heat equation is much less satisfactory than that for Laplace's equation, because the set of irregular boundary points is not necessarily polar, or even negligible. Therefore the behaviour of generalized solutions near irregular boundary points is more important. In this paper, we study such behaviour in the case of irregular points that are also cofine boundary points, that is, boundary points in the fine topology for the adjoint heat equation. We show that cofine limits exist and coincide with the values of the given boundary function at many such points, and characterize the points where this occurs in terms of both barriers and zero limits of the Green function. It is natural to call such points 'cofine regular'. Earlier, Bauer [3] and Doob [6], p. 358, have studied the existence of fine limits (that is, limits in the fine topology for the heat equation itself) of generalized solutions at irregular boundary points, but our results differ from theirs in both the topology and the fact that the limits assumed are the values of the given boundary function.

Non-polar sets of irregular boundary points that are also cofine boundary points, commonly occur within a single characteristic hyperplane. However, such a phenomenon does not occur if cofine irregular points are considered instead of (Euclidean) irregular ones. We prove this using a new reduction (or balayage) operator, which is introduced in Section 3. There Theorem 2 establishes the most important properties, and Theorem 3 uses these to prove a new result on the cofine boundary behaviour of greatest thermic minorants, which easily implies the existence of zero cofine limits of potentials (and hence of Green functions). Section 2 is devoted to proving a result which was stated and given an erroneous proof by Doob in [6]. In fact, we give a slight extension of the result, which is required for the proof of Theorem 2. Section 4 is where the results on cofine limits of generalized solutions are established. The methods are adaptations of classical techniques.

Throughout this paper, D denotes an arbitrary open subset of real Euclidean space $R^{n+1} = \{(x, t): x \in R^n, t \in R\}$. A typical point will be denoted by

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1980 Mathematics Subject Classification 31 B 35, 35K05.