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Linearized oscillations for difference equations

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1. Introduction and preliminaries

We obtain necessary and sufficient conditions for the oscillation of all solutions of the nonlinear difference equation

$$x_{n+1} - x_n + f(x_{n-k_1}, \dots, x_{n-k_m}) = 0, \qquad n = 0, 1, 2, \dots \tag{1}$$

in terms of the oscillation of all solutions of an associated linear difference equation.

Let N denote the set of nonnegative integers $\{0, 1, \dots\}$. Throughout this paper we will assume that

$$k_1, k_2, \dots, k_m \in N, \qquad f \in C[R^m, R] \tag{2}$$

and

$$\left.\begin{array}{ll} f(u_1, \dots, u_m) \geq 0 & \text{for } u_1, \dots, u_m \geq 0, \\ f(u_1, \dots, u_m) \leq 0 & \text{for } u_1, \dots, u_m \leq 0, \\ f(u, \dots, u) = 0 & \text{if and only if } u = 0. \end{array}\right\} \tag{3}$$

We will also assume that the following hypothesis holds:

(H) *There exists* $\delta > 0$ *such that* f *has continuous first partial derivatives,* $D_i f$, *for all* $u_1, \dots, u_m \in [-\delta, \delta]$ *such that*

$$D_i f(0, \dots, 0) = p_i \qquad \textit{for} \quad i = 1, \dots, m \tag{4}$$

with

$$p_1, \dots, p_m \in (0, \infty) \qquad \textit{and} \qquad \sum_{i=1}^{m} (p_i + k_i) \neq 1. \tag{5}$$

Furthermore,

either
$$f(u_1, \dots, u_m) \leq \sum_{i=1}^{m} p_i u_i \qquad \textit{for} \quad u_1, \dots, u_m \in [0, \delta] \tag{6}$$

or
$$f(u_1, \dots, u_m) \geq \sum_{i=1}^{m} p_i u_i \qquad \textit{for} \quad u_1, \dots, u_m \in [-\delta, 0]. \tag{6'}$$

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