HIROSHIMA MATH. J.
22 (1992), 57–93

The explicit representation of the determinant of Harish-Chandra's C-function in $SL(3, R)$ and $SL(4, R)$ cases

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(Received April 10, 1990)

§ 1. Introduction

Let G be a semisimple Lie group with finite center, K a maximal compact subgroup of G. Let θ be the Cartan involution of G fixing K. Let P be a cuspidal parabolic subgroup and $P=MAN$ its Langlands decomposition.

Let $\pi_{P,\sigma,\nu} = \mathrm{ind}_{MAN}^{G}\sigma \otimes \nu \otimes 1$ (σ in $\hat{M}$, ν a character of A) be the representation of the generalized principal series induced from P to G and $H^{P,\sigma,\nu}$ be its representation space. Then the operator $A(\bar{P}:P:\sigma:\nu)$ defined by the integral

$$(A(\bar{P}:P:\sigma:\nu)f)(x) = \int_{\bar{N}} f(x\bar{n})d\bar{n}\,, \qquad (f \in H^{P,\sigma,\nu})$$

is an intertwining operator between $\pi_{P,\sigma,\nu}(g)$ and $\pi_{\bar{P},\sigma,\nu}(g)$ $(g \in G)$, where $\bar{P} = \theta P$.

In the following we assume that P is a minimal parabolic subgroup of G. For γ in $\hat{K}$ we denote by $H_{\gamma}^{P,\sigma,\nu}$ the γ-isotypic component of $H^{P,\sigma,\nu}$. Let V^{γ} and H^{σ} be the representation spaces of γ and σ respectively. Following Wallach [11], we consider the bijective map $v \otimes A \to L_P(A, v, \nu)$ $(v \in V^{\gamma}, A \in \mathrm{Hom}_M(V^{\gamma}, H^{\sigma}))$ from $V^{\gamma} \otimes \mathrm{Hom}_M(V^{\gamma}, H^{\sigma})$ to $H_{\gamma}^{P,\sigma,\nu}$, where $L_P(A, v, \nu)$ is defined by

$$L_P(A, v, \nu)(kan) = e^{-(\nu+\rho)(\log a)}A(\pi_{\gamma}(k^{-1})v)\,, \qquad (k \in K, a \in A, n \in N)\,,$$

and the operator defined by the integral

$$B_{\gamma}(\bar{P}:P:\nu) = \int_{\bar{N}} \pi_{\gamma}(\kappa(\bar{n}))^{-1}e^{-(\nu+\rho)(H(\bar{n}))}d\bar{n}\,.$$

Then the operator $B_{\gamma}(\bar{P}:P:\nu)$ satisfies

$$A_{\gamma}(\bar{P}:P:\sigma:\nu)L_P(A, v, \nu) = L_{\bar{P}}(A \circ B_{\gamma}(\bar{P}:P:\nu), v, \nu)\,.$$

Moreover, $B_{\gamma}(\bar{P}:P:\nu)$ commutes with $\pi_{\gamma}(m)$ $(m \in M)$ and we can restrict B_{γ} to V_{σ}^{γ}, V_{σ}^{γ} denoting the σ-isotypic component of V^{γ}. We denote by B_{γ}^{σ} the restriction of B_{γ} to V_{σ}^{γ}. Wallach [11] has shown that $B_{\gamma}(\bar{P}:P:\nu)$ is holo-