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Legendre character sums

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1. Introduction

Character sum analogue of certain orthogonal polynomials were defined by Evans [1], but he mainly studied the properties of Hermite character sums. On the other hand, Greene [2] studied character sum analogue of hypergeometric series and has shown its usefulness. Classically it is well known certain orthogonal polynomials are deeply connected with the hypergeometric series. So it is natural to study the character sum analogue of orthogonal polynomials systematically and relate them to Greene's work.

In this paper, we consider the analogues of Legendre polynomials and its relation to Greene's hypergeometric character sums.

Throughout this paper we use the same notation as in Evans [1] and Greene [2].

Elementary formulae of Gauss and Jacobi sums which we will make use of are

$$G(N)G(\bar{N}) = qN(-1) \qquad (N \neq \varepsilon),\ G(\varepsilon) = -1 \tag{1.1}$$

$$J(M, N) = \frac{G(M)G(N)}{G(MN)} \qquad (MN \neq \varepsilon) \tag{1.2}$$

$$J(N, \bar{N}) = -N(-1) \qquad (N \neq \varepsilon),\ J(\varepsilon, \varepsilon) = q - 2 \tag{1.3}$$

$$G(\chi^2)G(\phi) = \chi(4)G(\chi)G(\chi\phi) \qquad (\phi^2 = \varepsilon) \tag{1.4}$$

2. Legendre character sums

In this section, we show that Legendre character sums have the similar properties as Legendre polynomials.

Evans [1] defined *Legendre character sums* as follows.

For any character N of $F_q^{\times}$,

$$P_N(x) = \frac{1}{q}\sum_u N(u)\,\phi(1 - 2xu + u^2).$$

We can show Legendre character sums have the generating function. (See, for example, [5] p. 157)