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Estimates of the euclidean span for an open Riemann surface of genus one

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1. Introduction

Every open Riemann surface of finite genus can be embedded conformally into a compact Riemann surface of the same genus. On the basis of [5], M. Shiba and K. Shibata gave in [7] a new proof of this classical theorem, and introduced the notion of hydrodynamic continuation. Their proof was of global character.

In [6] M. Shiba studied the set of compact continuations of an open Riemann surface of genus one in detail. He proved, among others, that the moduli set of compact continuations of a fixed marked open Riemann surface R of genus one is precisely a closed disk (or a point) in the upper half plane and that there is a bijection between the boundary of the closed disk and the set of hydrodynamic continuations of R. These results considerably improved Heins' result [2, Theorem 2]. The euclidean (resp. noneuclidean) diameter of the closed disk is called the euclidean (resp. hyperbolic) span for R (cf. Shiba-Shibata [8]). These spans represent the size of the ideal boundary of R. For example, the hyperbolic (or euclidean) span vanishes if and only if $R \in O_{AD}$ (see [6, Theorem 6]).

It seems that only few quantitative results about the moduli set are known. Shiba-Shibata [8] have calculated, using Jacobi's elliptic functions, the hyperbolic span explicitly for a strongly symmetric marked torus with a horizontal slit, and applied the formulae to estimate the hyperbolic span for an arbitrary marked torus with a horizontal slit. The results are rather complicated, however.

In this paper we consider an open Riemann surface (of genus one) of the form $R = \tilde{R}/G$, where G is a group generated by two translations of $\mathbb{C}$ and $\tilde{R}$ is a G-invariant domain of $\mathbb{C}$. By applying the length-area method we will give simple estimates of the euclidean span for R.

In the next section, after summarizing Shiba's results [6], we will state our main results. One of the hydrodynamic continuations of R has the smallest (normalized) area among the compact continuations of R. In §3 we will characterize the area in terms of the moduli of ring domains on R and