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An algorithm for computing multivariate isotonic regression

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1 Introduction

Isotonic regression theory plays a key role in the field of order restricted statistical inference. Most of the theory related to this which appeared in the literature prior to the seventies are reviewed with a historical background in the seminal book of Barlow, Bartholomew, Bremner and Brunk [1]. Since then extensive research has been done in this field filling many gaps in the theory and most of them are contained in the recently published book of Robertson, Wright and Dykstra [7]. A multivariate generalization of isotonic regression including the multivariate extensions of well-known Bartholomew's $\bar{\chi}^2$ and $\bar{E}^2$ is given by Sasabuchi, Inutsuka and Kulatunga [8]. This theory enables us to study statistical inference for ordered vector-valued parameters or sets of ordered parameters. Some of them are discussed by Kulatunga and Sasabuchi [5] and Kulatunga, Inutsuka and Sasabuchi [4]. An algorithm for the computation of bivariate isotonic regression is also demonstrated in Sasabuchi et al.'s paper [8]. This algorithm involves iterative computation of univariate isotonic regression. The main purpose of this paper is to present a multivariate generalization of the algorithm described for the bivariate case.

The definition of multivariate isotonic regression and some important results are stated in section 2. In section 3 we describe the multivariate generalization of the algorithm. Some theorems on convergence of the algorithm are given in section 4.

2 Definitions and basic theorems

First in our notation, we state the definition of univariate isotonic regression (see, Robertson et al. [7, p. 25]).

Let $K = \{1, \dots, k\}$ be a finite set on which a partial order $\ll$ is defined. The partial order on K may or may not be the natural order among positive integers $1 \ll 2 \ll \cdots \ll k$, which is called the simple order.

DEFINITION 2.1. A real vector $(\theta_1, \dots, \theta_k)$ is said to be *isotonic* with respect to the partial order $\ll$, if μ, $\nu \in K$ and $\mu \ll \nu$ imply $\theta_\mu \leq \theta_\nu$.