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Nonoscillatory solutions of systems of neutral differential equations

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1. Introduction

We consider the following system of neutral differential equations of the form

$$(1_\mu) \qquad \frac{d^n}{dt^n}[x_i(t) + (-1)^\mu a_i(t)x_i(h_i(t))] = \sum_{j=1}^{N} P_{ij}(t)f_{ij}(x_j(g_{ij}(t))),$$

$i = 1, 2, \ldots, N,\ N \geq 2,\ n \geq 1,\ \mu \in \{0, 1\},\ t_0 \geq 0$, where

(a) $a_i\colon [t_0, \infty) \to (0, \beta_i],\quad 0 < \beta_i < 1,\quad h_i, g_{ij}, P_{ij}\colon [t_0, \infty) \to R$, and $f_{ij}\colon R \to R$, $i, j = 1, 2, \ldots, N$ are continuous functions

(b) $h_i(t) \leq t,\ t \geq t_0,\ \lim_{t\to\infty} h_i(t) = \infty,\ \lim_{t\to\infty} g_{ij}(t) = \infty,\ i, j = 1, \ldots, N$;

(c) $uf_{ij}(u) > 0$ for $u \neq 0,\ i, j = 1, 2, \ldots, N$;

(d) $\lim_{t\to\infty} a_i(t) = a_{i0} \in [0, \beta_i],\ i = 1, 2, \ldots, N$.

Let $t_1 > t_0$. Denote

$$t_2 = \min\{\inf_{t\geq t_1} h_i(t),\ \inf_{t\geq t_1} g_{ij}(t);\ i, j = 1, \ldots, N\}.$$

A function $X = (x_1, \ldots, x_N)$ is a solution of (1_μ), if there exists a $t_1 \geq t_0$ such that $X(t)$ is continuous on $[t_2, \infty)$, $x_i(t) + (-1)^\mu a_i(t)x_i(h_i(t))$, $(i = 1, \ldots, N)$ are n-times continuously differentiable on $[t_1, \infty)$ and X satisfies (1_μ) on $[t_1, \infty)$.

A solution $X = (x_1, \ldots, x_N)$ of (1_μ) is nonoscillatory if there exists an $a \geq t_0$ such that every its component is different from zero for all large $t \geq a$.

The asymptotic properties of nonoscillatory solutions of neutral differential equations with variable coeficients and systems of nonlinear differential equations with deviating arguments have been studied for examle in [1–3, 4, 6, 7].

In this paper we prove the existence of nonoscillatory solutions of the system (1_μ) which approach to nonzero constant vectors as $t \to \infty$.

Denote

$$(2) \qquad H_i(0, t) \equiv t,\ H_i(k, t) = H_i(k-1, h_i(t)),\ i = 1, \ldots, N,\ k = 1, 2, \ldots,$$