

## A linearization of $S(U(1) \times U(2)) \setminus SU(1, 2)$ $\sigma$ -model

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(Received September 18, 1990)

(Revised March 16, 1991)

### Introduction

The stationary axisymmetric Einstein-Maxwell equations have been closely investigated by many mathematicians and physicists since Weyl solved completely the static axially symmetric class of vacuum gravitational fields in 1917 [12]. In particular this subject experienced a drastic scene in the years 1978–1980 with the development of solution generating methods. Geroch has found that each given stationary axisymmetric solution of the Einstein field equations is accompanied by an infinite family of potentials [5]. This fact has led to the fact that there exists an action of some infinite dimensional group, so called Geroch group, on the space of (local) solutions. This symmetry is sometimes called the hidden symmetry. He also conjectured that this action is transitive up to gauge transformations. This conjecture, called the Geroch conjecture, was proved affirmatively by I. Hauser and F. J. Ernst [7]. They derived a non-linear differential equation for matrix-valued functions from the field equations and generalized their results to the case which has  $N$  Abelian gauge potentials interacting with the gravitational field. In [3] H. Doi and K. Okamoto generalized the results of [7] to the case that the field equations take their values in an affine symmetric space, so that a “Kac-Moody” Lie group acts transitively on the space of solutions.

The purpose of this paper is to give a recipe for constructing solutions, following the method explored by P. Breitenlohner and D. Maison [1], who dealt with the Einstein equations there. But in this paper we treat the gravitational field interacting with electro-magnetic fields. The essential point of [1] was that the Ernst equations derived from the Einstein field equations [4] should be formulated as a  $\sigma$ -model which takes its value in an affine symmetric space or a Riemannian symmetric space. (The former is transformed into the latter by Kramer-Neugebauer transformation.) A linearization is, as in [1], carried out after introducing a 1-form taking its value in some Lie algebra with a spectral parameter.

In this paper we discuss in the category of formal power series. We treat only a  $\sigma$ -model with values in a Riemannian symmetric space which is derived from the Einstein-Maxwell field equations. In order to do that, we have to