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Applications of fractional calculus to ordinary and partial differential equations of the second order

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Abstract. In this paper, applications of the fractional calculus to the form

$$(z-a)(z-b)\varphi_2 + (C+Dz)\varphi_1 + E\varphi = f \qquad (z \neq a, z \neq b)$$

and the partial differential equation

$$\frac{\partial^2 \mu}{\partial z^2}(z-a)(z-b) + (C+Dz)\frac{\partial \mu}{\partial z} + \delta \cdot \mu(z, t) = A\frac{\partial^2 \mu}{\partial t^2} + B\frac{\partial \mu}{\partial t} \qquad (z \neq a, z \neq b)$$

are discussed.

§0. Introduction

Fractional calculus is a very useful and simple means in obtaining particular solutions to certain non-homogeneous linear differential equations. The solutions of linear ordinary differential equations of the Fuchs type [1]–[7], Gauss type [8, 9] and Laguerre's type [10] obtained by K. Nishimoto, S. L. Kalla, H. M. Srivastava, S. Owa, and S. T. Tu, are but a few important discoveries stemming from these researches. Now, we begin with the statement of the following definition of the fractional calculus (fractional integrals and fractional derivatives) given by Nishimoto 1976.

DEFINITION. If $f(z)$ is a regular function and it has no branch point inside C and on $C(C = \{C_-, C_+\}$, C_- is an integral curve along the cut joining two points z and $-\infty + i\ \mathrm{Im}(z)$, and C_+ is an integral curve along the cut joining two points z and $\infty + i\ \mathrm{Im}(z))$, $D = \{D_-, D_+\}$, D_- is a domain surrounded by C_-, D_+ is a domain surrounded by C_+,

$$f_\nu = {}_Cf_\nu(z) = \frac{\Gamma(\nu+1)}{2\pi i}\int_C \frac{f(\zeta)}{(\zeta - z)^{\nu+1}}\,d\zeta \qquad \begin{pmatrix} \Gamma\text{: Gamma function} \\ \nu \neq -1, -2, \dots \end{pmatrix}$$

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