HIROSHIMA MATH. J.
23 (1993), 15–61

Admissibility of difference approximations for scalar conservation laws

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(Received September 20, 1991)

Introduction

This paper is concerned with difference approximations for initial value problems for scalar conservation laws of the form

$$
\begin{cases} u_t + f(u)_x = 0, & x \in \boldsymbol{R},\ t > 0, \\ u(x, 0) = u_0(x), & x \in \boldsymbol{R}, \end{cases} \tag{0.1}
$$

where $u = u(x, t)$ is an unknown function, the *flux function* $f\colon \boldsymbol{R} \to \boldsymbol{R}$ is a function of C^1-class and the initial function u_0 is a bounded measurable function of bounded variation. Various types of difference approximations have been investigated by many authors. We refer the readers to, for instance, [2, 4, 5, 6, 7, 10, 14, 15, 16, 20, 21, 23, 24, 28, 29].

In this paper we study difference approximations *in viscous form*, namely,

$$
\begin{aligned} u_i^{n+1} = u_i^n &- \frac{\lambda}{2}\{f(u_{i+1}^n) - f(u_{i-1}^n)\} \\ &+ \frac{\lambda}{2}\{a_{i+\frac{1}{2}}^n(u_{i+1}^n - u_i^n) - a_{i-\frac{1}{2}}^n(u_i^n - u_{i-1}^n)\}, \qquad n, i \in \boldsymbol{Z},\ n \geq 0, \end{aligned} \tag{0.2}
$$

where initial values u_i^0 are given data and $\lambda = \dfrac{\Delta t}{\Delta x}$ is a fixed constant, Δx the mesh size in space-direction and Δt in time-direction. Each of $\lambda a_{i+\frac{1}{2}}^n$ is called a numerical viscosity coefficient [8, 24, 29]. For the initial values u_i^0, we assume that

$$
m \leq u_i^0 \leq M, \qquad i \in \boldsymbol{Z}, \tag{0.3}
$$

for some constants m and M both independent of Δx, and

$$
\sup_{\Delta x} \sum_{i \in \boldsymbol{Z}} |u_{i+1}^0 - u_i^0| < +\infty. \tag{0.4}
$$

On the mesh ratio λ, we impose so called CFL condition