HIROSHIMA MATH. J.
**23** (1993), 417–448

# Numerical treatment on the behavior of interfaces in oil-reservoir problems

Tatsuyuki NAKAKI

(Received January 18, 1992)

## 1. Introduction

Nonlinear problems arising in industrial mathematics have been extensively investigated for a long time. In the field of the reservoir engineering, there appear interesting phenomena, which will be explained below. In order to recover part of the remaining oil from wells (called production wells), it is used the way that one injects a water into another wells (injection wells), which are located around the reservoir, so that the water pushes the oil toward the production wells, and then the oil can be recovered. In this process, two immiscible fluids, water and oil, can be regarded as separated by a sharp interface during the penetration of water into oil. By laboratory studies, it is already known that the interface is unstable; small perturbations in the interface grow up (see [2] or Fig. 18). This phenomenon is called the *fingering instability*. When the area of water reaches the production wells, pockets of by-passed oil are created, and water is produced from production wells, which is economically unfavorable.

Similar phenomenon is observed in the Hele-Shaw cell involving flow in thin gaps [14], and also observed in the flow caused by Rayleigh-Taylor instability [16]. In the former, the interface between two immiscible fluids becomes unstable and forms fingers, when a less viscous fluid moves slowly between two parallel horizontal plates separated by a thin gap, in which a more viscous fluid is filled. In the latter, the interface also exhibits a fingering pattern, when a lighter fluid fails to support a heavier fluid against the action of gravity.

Especially, to analyze the oil recovery process, mathematical models have been proposed and studied from numerical points of view, where the capillary pressure between water and oil is ignored ([8], [5], [6], [18] and the references therein). In particular, the Buckley-Leverett equations, which consist of the one hyperbolic equation and the two elliptic equations, are well known in the petroleum literature. Glimm et al [8] show some numerical simulations for these equations, and indicate the occurrence of the fingering instability. Similar simulations are done in [5], [6] and [18], where the interface with a