HIROSHIMA MATH. J.
23 (1993), 249–303

The conformal factor and a central extension of a formal loop group with values in $PSL(2, \mathbf{R})$

Ryuichi SAWAE

(Received January 16, 1992)

Introduction

The main objective of this paper is to investigate a relation between a central extension of the Hauser group, which would be a subgroup of "Geroch group", and the conformal factor of the Einstein vacuum filed equations in a 2-dimensional reduction. The conformal factor is considered to be τ-function (for example see [10][17]) in case of the Einstein vacuum filed equations. As far as the author knows, approaches in this directions were undertaken by P. Breitenlohner and D. Maison [1], K. Okamoto [16] and B. Julia [9].

On the other hand, "solution generating methods" of the stationary axisymmetric Einstein vacuum equations and the Einstein-Maxwell equations have been drastically investigated since Geroch [6] had found that each given stationary axisymmetric solution of the Einstein field equations are accompanied by an infinite family of potentials. Geroch's observation has led to W. Kinnersley's formulation [11] and to the fact that there exists an action of some infinite dimensional group, so called Geroch group, on the space of solutions. Geroch conjecture was proved affirmatively by I. Hauser and F. J. Ernst [8] following Kinnersley's formulation. In [2] H. Doi and K. Okamoto generalized the results of [8] to the case that the field equations take their values in an affine symmetric space, so that a "Kac-Moody" Lie group acts transitively on the space of solutions. However the action of center of the "Kac-Moody" Lie group was trivial. For another formulation and discussions, for example, see Y. S. Wu and M.L. Ge [21].

In the previous paper [7] (cf. [3][4]), a σ-model with values in an affine symmetric space, that is, $S(U(1) \times U(2)) \backslash SU(1, 2)$ was formulated with a linearization method explored by P. Breitenlohner and D. Maison [1] and a formal loop group method established by K. Takasaki [18]. And a recipe for constructing solutions was given there, which gives the gravitational field interacting with electro-magnetic fields. But no conformal factor was dealt with.

In this paper, a σ-model with values in $K \backslash PSL(2, \mathbf{R})$ is treated with the