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Borel-Weil theory and Feynman path integrals on flag manifolds

Takashi HASHIMOTO, Kazunori OGURA,
Kiyosato OKAMOTO and Ryuichi SAWAE*)

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0 Introduction

In [6] we computed path integrals on coadjoint orbits of the Heisenberg group, $SU(1, 1)$ and $SU(2)$ etc.. As to the Heisenberg group, we succeeded in computing the path integrals for complex polarizations as well as real polarizations.

For the complex polarizations of $SU(1, 1)$ and $SU(2)$, however, we found it difficult to carry out the computation of path integrals, so that we computed the path integrals without Hamiltonians. Soon after we encountered difficulty of divergence of the path integrals along the method in [6].

For the complex polarizations of $SU(1, 1)$ and $SU(2)$, by taking the operator ordering into account and then regularizing the path integrals by use of the explicit form of the integrand, we computed the path integrals with Hamiltonians in [7].

In this paper, we shall give an idea how to regularize the path integrals for complex polarizations of any connected semisimple Lie group G which contains a compact Cartan subgroup T and shall show, along this idea, that the path integral gives the kernel function of the irreducible unitary representation of G realized by Borel-Weil theory.

Our idea is roughly explained as follows. Let $\mathfrak{h}$ be the Lie algebra of T and $\mathfrak{h}^{\mathbf{C}}$ the complexification of $\mathfrak{h}$. Denote by $\mathfrak{n}^+$ and $\mathfrak{n}^-$ the Lie algebras spanned by the positive root vectors and the negative root vectors, respectively. For any integral form Λ on $\mathfrak{h}^{\mathbf{C}}$ we denote by ξ_Λ the holomorphic character of $T^{\mathbf{C}}$ defined by Λ and by L_Λ the associated holomorphic line bundle on the flag manifold G/T. Let π_Λ be the irrducible unitary representation of G on the Hilbert space of all square integrable holomorphic sections of L_Λ which is realized by the Borel-Weil theorem.

Put $\lambda = \sqrt{-1}\Lambda$. Then for any element Y of the Lie algebra of G, the Hamiltonian on the flag manifold G/T is defind by

$$\begin{aligned} H_Y(g) &= \langle \mathrm{Ad}^*(g)\lambda, Y\rangle \\ &= \sqrt{-1}\Lambda(\mathrm{Ad}(g^{-1})Y). \end{aligned}$$