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Classification and uniqueness of positive solutions of $\Delta u + f(u) = 0$

Dedicated to Professor Takaŝi Kusano on his 60th birthday

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Abstract

Given $0 \leq \theta < \infty$. In this paper, we classify the solutions of the initial value problem

$$(*) \qquad \begin{cases} u''(r) + \dfrac{m}{r}u'(r) + f(u(r)) = 0 \text{ on } (\theta, R(\xi)), \\ u(\theta) = \xi > 0 \text{ and } u'(\theta) = 0, \end{cases}$$

where f is locally Lipschitz on $(0, \infty)$ and there exist two positive constants α, β such that $f(u) < 0$ on $(0, \alpha)$, $f(u) > 0$ on (α, ∞) and $F(\beta) > 0$. Here $R(\xi) := \sup\{r \in (\theta, \infty) | u(s) > 0 \text{ for all } s \in [\theta, r)\}$ and $F(u) := \int_0^u f(s)ds$ for $u \geq 0$. Moreover, we establish an existence-uniqueness theorem of a solution for equation $(*)$ satisfying $u'(0) = 0$ and $\lim_{r \to \infty} u(r) = 0$.

1. Introduction

Let $\theta \in [0, \infty)$ be given and $\boldsymbol{R}^n (n \geq 2)$ denote the usual n-dimensional Euclidean space. Consider the following two problems:

$$(\mathrm{I}_1) \qquad \begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega(R(\xi)), \\ \dfrac{\partial u}{\partial n} = 0 & \text{if } |x| = \theta, \\ u(x) = \xi > 0 & \text{if } |x| = \theta; \end{cases}$$

$$(\mathrm{I}_2) \qquad \begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega(\infty), \\ \dfrac{\partial u}{\partial n} = 0 & \text{if } |x| = \theta, \\ \lim\limits_{|x| \to \infty} u(x) = 0, \end{cases}$$

where $R(\xi) := \sup\{r \in (\theta, \infty) | u(x) > 0 \text{ for } \theta \leq |x| < r\}$ and

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