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Conformal mapping of geodesically slit tori and an application to the evaluation of the hyperbolic span

M. SHIBA and K. SHIBATA

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Introduction

Throughout this paper an open Riemann surface of genus one is called an *open torus*; it will be called a *geodesically slit torus* if it arises from a *symmetric* torus by removing a *single* segment lying on the axis of symmetry. For the precise definitions, see Section 1.

Our aim is to find a conformal mapping of a geodesically slit torus onto another and to study the spans of such an open torus. More specifically, we consider two geodesically slit tori; one has a slit along a geodesic homotopic to a longitude and the other has a slit along a geodesic homotopic to a meridian. Our first problem is then to give criteria for such slit tori to be conformally equivalent. A conformal mapping between them, if any, will be constructed by means of Jacobi's elliptic functions. The method employed here is thus classical and the idea is also simple, but we would like to remind the reader of the fact that the corresponding classical problem of studying conformal mapping between a horizontal slit rectangle and a vertical slit rectangle is more difficult (cf. [5]). Slit tori can be sometimes dealt with more easily than slit rectangles. The reason is that in the case of tori we can normalize the slit so as to lie over the boundary of a fundamental region.

The second problem is to evaluate the hyperbolic span of a geodesically slit torus. We also find some estimates of the euclidean span of general open tori. These spans have been introduced in [8] and [9], as generalizations of the Schiffer span of plane domains.

We see that the hyperbolic span of a geodesically slit torus is exactly expressed by means of the complete elliptic integrals of the first kind. Some numerical examples will be also given. We finally observe that the cutting and pasting method yields all the tori from a single torus.

We remark that a theoretical treatment for plane rectangles was established by Jenkins ([3]), and its counterpart for open tori has been given in [8] and [9]. We also note that in [9] we define another span, the spherical span, of an open torus, and investigate the relation of these —euclidean, hyperbolic,