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Remark on the characterization of weighted Besov spaces via temperatures

Dedicated to Professor Makoto Ohtsuka on the occasion of his 70*th* birthday

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1. Introduction and statements of the results

The aim of this note is to complete the characterization of the weighted homogeneous Besov spaces by means of solutions of the heat equation on $\mathbf{R}_+^{n+1}$, which was initiated in [2]. We retain all the notations and terminologies in [1, 2] to which the reader is referred also for background and references to related works in the literature.

In [2, Theorem 1′] we proved that

$$\|f\|_{\dot{B}(s,w,p,q)} \le C\left(\int_0^\infty (t^{k-s/2}\|(\partial/\partial t)^k(W_t * f)\|_{p,w})^q \frac{dt}{t}\right)^{1/q} \tag{1}$$

for any $f \in \mathscr{S}'$, where $w \in A_\infty$, $0 < p < \infty$, $0 < q \le \infty$, $s \in \mathbf{R}$, k is a non-negative integer greater than $s/2$, $W_t(x) = (4t)^{-n/2}e^{-|x|^2/4t}$ is the Gauss-Weierstrass kernel on $\mathbf{R}_+^{n+1}$, and as usual, we use C, c, ... to denote positive constants whose values might change from one occurrence to the next one; they might depend on the parameters s, p, q, k, ..., but not on the distribution f. If a constant depends also on f, we use subscript to denote this dependence, e.g., C_f.

For the opposite direction to (1), in the same quoted theorem, we had to assume that w is furthermore in $\dot{\mathscr{M}}_d$. In this note we shall remove this restriction. Namely, we shall prove the following result.

Theorem. *If $f \in \dot{B}_{p,q}^{s;w}$ and k is a non-negative integer greater than $s/2 + \max(0, 1/p - 1, 1/q - 1)$, then there is a polynomial P such that*

$$\left(\int_0^\infty (t^{k-s/2}\|(\partial/\partial t)^k(W_t * (f - P))\|_{H(p,w)})^q \frac{dt}{t}\right)^{1/q} \le C\|f\|_{\dot{B}(s,w,p,q)}. \tag{2}$$

Remark. Though we have removed the restriction $w \in \dot{\mathscr{M}}_d$, we have introduced a new restriction on the range of k in the case either $p < 1$ or $q < 1$. This new restriction is, however, more satisfactory than the other, as it does not depend on the weight function w, and the result in the Theorem