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# Fractional powers of operators with polynomially bounded resolvent and the semigroups generated by them

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## 0. Introduction

The first result on fractional powers of a closed operator was obtained by S. Bochner [ 4 ]. In 1949, he constructed fractional powers of $-\Delta$, using essentially that the Laplacian $\Delta$ generates a bounded $C_0$-semigroup. E. Hille and R. Phillips (see [ 8 ] and [21]) took up this idea and defined fractional powers of the negatives of arbitrary generators of bounded $C_0$-semigroups.

In 1960, A. V. Balakrishnan [ 2 ], giving a new definition, extended the theory of fractional powers to closed operators $A$ for which the resolvent $R(\lambda, A) := (\lambda - A)^{-1}$ exists and satisfies

$$\|R(\lambda, A)\| \leq \frac{C}{|\lambda|}$$

on an open sector $\{\lambda \in \boldsymbol{C}: |\arg \lambda| < a\}$ for some $0 < a < \frac{\pi}{2}$. For such operators, other, by [ 14 ] equivalent definitions of fractional powers have been given, e.g. by T. Kato [10], H. Komatsu [12], H. W. Hövel and U. Westphal [ 9 ] and C. Martinez, M. Sanz and L. Marco [15].

Motivated by the examples below, we study in this paper fractional powers $(-A)^b$ $(b \in \boldsymbol{C})$ and the semigroups generated by their negatives (if any) in the case that the resolvent set $\rho(A)$ contains a closed sector $\Sigma(a) = \{\lambda \in \boldsymbol{C}: |\arg \lambda| \leq a\} \cup \{0\}$, and the resolvent satisfies

$$\|R(\lambda, A)\| \leq C(1 + |\lambda|)^n$$

for some $n \in \boldsymbol{N}_0$ and all $\lambda \in \Sigma(a)$.

EXAMPLES. (1) A closed, densely defined, linear operator $A$ is the generator of an integrated semigroup (see for example [ 11 ] or [18]) if and only if the resolvent of $A$ exists and is polynomially bounded on a right half plane $\{\lambda \in \boldsymbol{C}: \Re\lambda > \omega\}$. If in addition $[0, \infty) \subseteq \rho(A)$, then $A$ belongs to the class of operators we will discuss in this paper. The domains of the fractional powers of such generators are important in the study of the associated abstract Cauchy problem