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On links whose complements have the Lusternik-Schnirelmann category one

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1. Introduction

The Lusternik-Schnirelmann category $\operatorname{cat} X$ of a space X is the least integer n such that X can be covered by $n+1$ open subsets each of which is contractible to a point in X. In particular, $\operatorname{cat} X$ is a homotopy type invariant and $\operatorname{cat}(\bigvee_i S^{n_i}) = 1$ where $\bigvee$ stands for the one point union. We know that $\pi_1(X)$ is a free group if X is a manifold and $\operatorname{cat} X = 1$ [4], [7].

A locally flat knot (S^{n+2}, S^n) is topologically unknotted if and only if the category of its complement is one [14]. So, a smooth (or PL locally flat) knot (S^{n+2}, S^n) is unknotted if and only if $\operatorname{cat}(S^{n+2} - S^n) = 1$ when $n \neq 2$ ([12], [25] for $n \geq 4$, [21] for $n = 3$ and [18] for $n = 1$). We know also that there exists a smooth knot (S^{n+2}, S^n) whose complement is of category m with $2 \leq m \leq n+1$ for any n [15], [16].

A smooth (resp. PL locally flat or locally flat) m-component link L stands for m smoothly (resp. PL locally flatly or locally flatly) embedded disjoint n-spheres $L_1 \cup \cdots \cup L_m$ in S^{n+2}. A smooth (resp. PL locally flat or locally flat) m-component link is called trivial if it bounds m smoothly (resp. PL locally flatly or locally flatly) embedded disjoint $(n+1)$-disks; boundary if it bounds a Seifert manifold which consists of m disjoint compact smooth (resp. PL locally flat or locally flat) $(n+1)$-submanifolds with connected boundary. Let $N_i = N(L_i)(i = 1, \ldots, m)$ be the tubular neighborhoods of L_i which do not intersect each other. The compact manifold $E = S^{n+2} - \bigcup \operatorname{Int} N(L_i)$ with boundary $\partial E = \bigcup \partial N_i$ is called link exterior and has the homotopy type of the link complement $S^{n+2} - L$.

A smooth boundary link (S^{n+2}, L) is trivial if $\operatorname{cat}(S^{n+2} - L) = 1$ when $n \neq 2$ [11]. In particular, the complement $S^{n+2} - L$ of a smooth boundary link L has the homotopy type of $(\bigvee_m S^1) \vee (\bigvee_{m-1} S^{n+1})$ if $\operatorname{cat}(S^{n+2} - L) = 1$ when $n \neq 2$.

The purpose of this paper is to show the following Theorems 1 and 2. Note that any smooth or PL locally flat link is locally flat. So, Theorem 1 gives an alternative proof of the main theorem of [11] by unlinking criterion of boundary links due to Gutiérrez ([8] for $n \geq 4$ and use the splitting