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# Ergodic theorems for piecewise affine Markov maps with indifferent fixed points

Tomoki INOUE

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We study piecewise affine Markov maps with "indifferent fixed points", which are interesting with relation to intermittency. In the case that such a map $T$ has a Lebesgue-equivalent invariant $\sigma$-finite infinite measure $\mu$, we give ratio ergodic theorems which describe the limit value of the ratio of the sojourn time of the trajectory $\{T^k x\}_{k=0}^n$ in an interval $U_1$ with $\mu(U_1) = \infty$ to that in another interval $U_2$ with $\mu(U_2) = \infty$ for almost every $x$.

## 0. Introduction

For an interval map $T$, a fixed point $p$ is called *indifferent* if

$$\lim_{x \to p} T(x) = p \quad \text{and} \quad \lim_{x \to p} |T'(x)| = 1.$$

Maps with indifferent fixed points are related to physical type I intermittency (cf. [1], [10], [13]). Our interesting indifferent fixed point $p$ is a source, that is, $|T'(x)| > 1$ for almost every $x$ in the neighborhood of the fixed point $p$.

In Inoue's paper [4], the somewhat strange notion "weakly attracting repellors" is given, that is, a fixed point $p$ is called the *weakly attracting repellor* of an interval map $T$ if $p$ is unstable ($T^k x$ does not converge to $p$ for a.e. $x$) and if

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} f(T^k x) = f(p) \qquad \text{a.e. } x$$

for every continuous function $f$ on the interval. In [4] we gave some conditions for the existence of weakly atracting repellors for maps with only one indifferent fixed point.

In this paper we are going to study maps with at least one indifferent fixed point. A typical example of an interval map with indifferent fixed points is

$$T(x) = \begin{cases} x/(1-x) & \text{for } x \in [0, \frac{1}{2}) \\ (2x-1)/x & \text{for } x \in [\frac{1}{2}, 1], \end{cases}$$