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A theorem of Hardy-Littlewood and removability for polyharmonic functions satisfying Hölder's condition

Dedicated to Professor M. Nakai on the occasion of his sixtieth birthday

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Abstract

One of our aims in this note is to give an extension of a result of Hardy-Littlewood [3, Theorems 40 and 41] for holomorphic functions on the unit disc. In fact we show that a polyharmonic function u on the unit ball $\boldsymbol{B}$ satisfies Hölder's condition of exponent α, $0 < \alpha \leqq 1$, if and only if

$$|\operatorname{grad} u(x)| \leqq M(1 - |x|^2)^{\alpha - 1} \qquad \text{for any } x \in \boldsymbol{B}$$

by appealing to a mean-value inequality for polyharmonic functions.

Next we discuss removable singularities for polyharmonic functions u satisfying

$$|D^j u(x + y) + D^j u(x - y) - 2D^j u(x)| \leqq M|y|^{\alpha - k}$$

for all $x \in G$, y with $x \pm y \in G$ and j with $|j| = k$, where G is an open set in R^n and k is the nonnegative integer such that $k < \alpha \leqq k + 1$. Our goal is to derive a generalization of the recent result of Ullrich [12, Theorem 1].

1. Introduction

Let G be an open set in R^n. An infinitely differentiable function u on G is called polyharmonic of order m in G if $\Delta^m u = 0$ holds in G; we say that u is polyharmonic in G if it is polyharmonic of order m in G for some positive integer m. In case $0 < \alpha \leqq 1$, if a continuous function u on G satisfies

$$(1) \qquad |u(x) - u(y)| \leqq M|x - y|^\alpha \qquad \text{whenever } x, y \in G$$

for some constant M, then we say that u satisfies Hölder's condition of exponent α in G

In this paper let M denote various constants, whose value may change from one occurrence to the next. We denote by $\boldsymbol{B}$ the unit ball of R^n.

Our first aim in this paper is to prove

THEOREM 1. *Let u be a polyharmonic function on $\boldsymbol{B}$ and $0 < \alpha \leqq 1$. Then*