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A study of nonparametric estimation of error distribution in linear model based on L_1-norm

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1. Introduction

The field of nonparametric estimation has broadened its appeal in the last two decades with an array of new tools for statistical analysis. These new tools have offered sophisticated alternatives to the traditional parametric models in exploring large amounts of univariate or multivariate data without making any special distributional assumption. One of these tools is an estimation method of nonparametric density, which has become a prominent statistical research topic. Given identically distributed random variables $x_1, \cdots, x_n$ drawn from a population with density f, the aim is to construct an estimator of f without making any parametric assumption on the form of f. The pioneering papers might be due to Rosenblatt (1956) and Parzen (1962). Since the publication of these early papers, there has been a large amount of research on density estimation. In particular, theoretical and applied research on nonparametric density estimation has given noticeable influence on the related subjects, such as nonparametric regression, nonparametric discrimination, and so on, for the detail, see Alan (1991), Chao and Chai (1992), etc.

We consider a linear model

$$y_i = \boldsymbol{x}_i'\beta + e_i, \qquad i = 1, 2, \cdots, \tag{1.1}$$

where $\boldsymbol{x}_i'$s are $p(\geq 1)$-dimension known vectors and $\beta(\in R^p)$ is an unknown regression coefficient vector. The errors e_i's are assumed to be i.i.d. r.v.'s with a common unknown density function $f(x)$, and

$$E(e_1) = 0, \quad 0 < Var(e_1) = E(e_1^2) < \infty. \tag{1.2}$$

It is frequently assumed that e_1 has a normal distribution $N(0, \sigma^2)$ in usual regression analysis. Then, an estimator of β based on $(x_1, y_1), \cdots, (x_n, y_n)$ is obtained by the Least Squares method. The estimator, which is called the LSE of β, is defined as an unique solution $\hat{\beta}$ of the following minimization problem: