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Existence theorems for a neutral functional differential equation whose leading part contains a difference operator of higher degree

Yuichi KITAMURA and KUSANO Takaŝi

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0. Introduction

In this paper we are concerned with the problem of existence of solutions for a neutral functional differential equation of the form

$$D^n \Delta_\lambda^m x(t) + f(t, x(g(t))) = 0\,, \qquad t \geq t_0\,, \tag{A}$$

where D^n and Δ_λ^m stand, respectively, for the n-th iterate of the differential operator D and the m-th iterate of the difference operator Δ_λ defined by

$$Dx(t) = \frac{d}{dt}x(t) \quad \text{and} \quad \Delta_\lambda x(t) = x(t) - \lambda x(t - \tau)\,. \tag{0.1}$$

In case $\lambda = 1$ use is made of the symbol Δ instead of Δ_1, i.e.,

$$\Delta x(t) = x(t) - x(t - \tau)\,. \tag{0.2}$$

The conditions always assumed for (A) are as follows:

(0.3) (a) $m \geq 1$, $n \geq 1$, $\lambda > 0$, $\tau > 0$ and $t_0 > 0$;
(b) $g \in C[t_0, \infty)$, and $\lim_{t \to \infty} g(t) = \infty$;
(c) $f \in C([t_0, \infty) \times \boldsymbol{R})$, and

$$|f(t, x)| \leq F(t, |x|)\,, \qquad (t, x) \in [t_0, \infty) \times \boldsymbol{R}\,,$$

for some continuous function $F(t, u)$ on $[t_0, \infty) \times \boldsymbol{R}_+$, $\boldsymbol{R}_+ = [0, \infty)$, which is nondecreasing in u for each fixed $t \geq t_0$.

By a solution of (A) we mean a function $x \in C[T_x - m\tau, \infty)$ for some $T_x \geq t_0 + m\tau$ such that $\Delta_\lambda^m x(t)$ is n-times continuously differentiable and satisfies the equation on $[T_x, \infty)$. A solution of (A) is said to be oscillatory if it has an infinite sequence of zeros clustering at $t = \infty$; otherwise a solution is said to be nonoscillatory.

We observe that the associated unperturbed equation $D^n \Delta_\lambda^m x(t) = 0$ has the solutions