HIROSHIMA MATH. J.
**25** (1995), 19–41

# Nonlinear eigenvalue problem for a model equation of an elastic surface

Dedicated to Professor Takaŝi Kusano on his 60th birthday

Nobuyoshi FUKAGAI and Kimiaki NARUKAWA

(Received August 10, 1993)

## 1. Introduction

In this article we discuss the existence of nonzero weak solutions of the boundary value problem

$$-\gamma \operatorname{div}\left[\frac{(\sqrt{1+|\nabla u|^2}-1)^{\gamma-1}}{\sqrt{1+|\nabla u|^2}}\nabla u\right]=\lambda f(x, u) \qquad \text{in } \Omega \tag{1.1}$$

$$u \geq 0 \qquad \text{in } \Omega \tag{1.2}$$

$$u = 0 \qquad \text{on } \partial\Omega\,, \tag{1.3}$$

where $\gamma > 1$, $\Omega$ is a bounded domain in $\boldsymbol{R}^n$, $\nabla u$ denotes the gradient of $u$ and $\lambda$ is a positive parameter. In the case $\gamma = 1$, the equation (1.1) is the mean curvature equation or the capillary surface equation. When $n = 1$, this equation describes the equilibrium state of an elastic string yielding an exterior force $f(x, u)$. We give the derivation of the equation (1.1) for one dimensional elastic string in Section 2. The parameter $\lambda$ depends on a tension of the string. The purpose of this paper is to investigate the dependence between a weak solution $u_\lambda$ and parameter $\lambda$.

It is easy to see that solutions of (1.1), (1.3) correspond to critical points of the functional

$$I_\lambda[u] = \int_\Omega (\sqrt{1+|\nabla u|^2}-1)^\gamma dx - \lambda \int_\Omega F(x, u)dx \tag{1.4}$$

defined on the usual Sobolev space $W_0^{1,\gamma}(\Omega)$, where

$$F(x, u) = \int_0^u f(x, \xi)d\xi\,.$$

Under appropriate growth conditions on $F(x, u)$ we show the existence of a local minimizer of $I_\lambda$ in Section 3. Next we give a proof to obtain an unstable critical point of $I_\lambda$ by using the mountain pass lemma without Palais-Smale