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Oscillation properties of half-linear functional differential equations of the second order

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1. Introduction

This paper is devoted to the study of the oscillatory (and nonoscillatory) behavior of second order functional differential equations of the type

$$(|y'(t)|^{\alpha} \operatorname{sgn} y'(t))' + q(t)|y(g(t))|^{\alpha} \operatorname{sgn} y(g(t)) = 0 \tag{A}$$

for which the following conditions, collectively referred to as (H), are assumed to hold:

(a) α is a positive constant;
(b) $q(t)$ is a positive continuous function on $[a, \infty)$, $a \geqq 0$;
(c) $g(t)$ is a positive continuously differentiable function on $[a, \infty)$ such that $g'(t) > 0$ for $t \geqq a$ and $\lim_{t\to\infty} g(t) = \infty$.

By a solution of (A) we mean a function $y \in C^1(T_y, \infty)$, $T_y \geqq a$, which has the property $|y'|^{\alpha} \operatorname{sgn} y' \in C^1[T_y, \infty)$ and satisfies the equation for all sufficiently large t in $[T_y, \infty)$. Our attention will be restricted to those solutions which are nontrivial in the sense that $\sup\{|y(t)|: t \geqq T\} > 0$ for any $T > T_y$. Such a solution is said to be oscillatory if it has an infinite sequence of zeros clustering at ∞; otherwise it is said to be nonoscillatory. By definition, the equation (A) is oscillatory if all of its solutions are oscillatory and nonoscillatory otherwise.

The oscillation results for (A) to be proved in this paper are as follows.

THEOREM 1. *The equation* (A) *is oscillatory if*

$$\int_a^{\infty} q(t)dt = \infty. \tag{1.1}$$

THEOREM 2. *Suppose that*

$$\int_a^{\infty} q(t)dt < \infty \tag{1.2}$$