HIROSHIMA MATH. J.
25 (1995), 571–583

Oscillation criteria for hale-linear second order differential equations

Horng Jaan LI

(Received March 14, 1994)
(Revised May 31, 1994)

Abstract. Some oscillation criteria are given for second order nonlinear differential equation

$$[\Phi(u'(t))]' + c(t)\Phi(u(t)) = 0,$$

where $c(t)$ is a continuous function on $[0, \infty)$ and $\Phi: \mathbb{R} \to \mathbb{R}$ is defined by $\Phi(x) = |x|^{p-2}x$ with $p > 1$ a fixed real number. If $p = 2$, then these results improve earlier oscillation criteria of Wintner, Hartman, Kamenev and Philos.

1. Introduction

In the paper, we are concerned with the differential equation

$$\text{(E)} \qquad [\Phi(u'(t))]' + c(t)\Phi(u(t)) = 0, \qquad t \geq t_0,$$

where $c(t)$ is a continuous function on $[t_0, \infty)$ and $\Phi(s)$ is a real-valued function defined by $\Phi(s) = |s|^{p-2}s$ with $p > 1$ a fixed real number. If $p = 2$, then equation (E) reduces to the linear differential equation

$$(\mathrm{E}_1) \qquad u''(t) + c(t)u(t) = 0.$$

By a solution of (E) we mean a function $u \in C^1[t_0, \infty)$ such that $\Phi(u') \in C^1[t_0, \infty)$ and that satisfies (E). In [5], Pino established the existence, uniqueness and extension to $[t_0, \infty)$ of solutions to the initial value problem for (E). We will say that a nontrivial solution u of (E) is oscillatory if it has arbitrary large zeros, and otherwise it is nonoscillatory. Equation (E) is oscillatory if all its solutions are oscillatory.

Wintner [6] showed that equation (E_1) is oscillatory if

$$\lim_{t \to \infty} \frac{1}{t} \int_{t_0}^{t} \int_{t_0}^{s} c(\xi)\, d\xi\, ds = \infty.$$

Hartman [2] prove that the limit cannot be replaced by the upper limit in the above assumption and that the condition